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Original Research Article

Derivation of Mathematical Model for Micromotion and Micro-Displacement at the Bone-Implant Interface Using Spring Analysis

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Abstract: One of the major factors that determine the success of hip replacement is the primary stability which is the function of the micromotion on the bone-implant interface. Failure of hips replacement may arise from excessive motion at the implant-bone interface under the weight bearing loads. Minimizing the micromotion of the cementless prosthetic components is a key requirement for obtaining bone in-growth. If the initial movement is excessive, bone in-growth into the porous surface will not occur. Few experimental studies are available on implant micromotion largely due to difficulty of simulating loads in-vitro and in-vivo. Due to this reason, this research derived a theoretical model that relates the micro-velocity at which the implant moves down in the bone at the implant-bone surface at a specific time, the axial force applied on the head of the implant and the stiffness of the implants and the bones. The implant-bone interface (fibrous tissues) was taken as elastic surface that obeys Hooke's law using spring analysis. Here, the displacement of the implant equals the micromotion depending on the stiffness of both cortical and trabecular (cancellous) bones. When the implant is axially loaded, due to the elastic modulus of the bone when compared to that of the implant (stainless steel), the deformation of the implant is neglected. That is, Ei >> (E_C,E_T) where Ei ,E_C and E_T are the elastic moduli of the implant, cortical part of the bone and the trabecular part of the bone respectively.

Keywords: Cortical bone, Micromotion, Micro-displacement, Implant-bone interface, Spring stiffness.

INTRODUCTION

In artificial joint replacement the loosening of implants is a major reason for clinical failure. Two principal types of implants exist: cemented and uncemented implants. Generally, uncemented implants are surface treated in order to achieve bony anchorage. The knowledge about bone ingrowth (osseointegration) is important to improve the long-term stability of implants, but it is hard to measure in vivo [1]. Bone material presents a complex behaviour involving heterogeneous and anisotropic mechanical properties. Moreover, bone is a living tissue, therefore its microstructure and mechanical properties evolve with time, in a process called bone remodelling. This phenomenon has been studied from a long time, and there are many numerical models that have been formulated in this sense to predict the density distribution in various bones, mainly in the femur [12]. Therefore, the use of mathematical model for determination of micromotion and micro-displacement will be of utmost importance.

Micromotion and Primary Stability

The most commonly reported complications related to cementless hip stems are loosening and thigh pain; both of these have been attributed to high levels of relative micromotion at the bone–implant interface due to insufficient primary fixation. Primary fixation is believed by many to rely on achieving a sufficient interference fit between the implant and the bone. However, attempting to achieve a high interference fit not infrequently leads to femoral canal fracture either intra-operatively or soon after. The stability of prosthesis in the host bone is an important factor for the success of clinical surgery [2].

Failure of hips replacement may arise from excessive motion at the implant-bone interface under the weight bearing loads. Minimizing the micromotion of the cementless prosthetic components is a key requirement for obtaining bone in-growth. Cementless arthroplasty uses mechanical press fit contacts between the implant and the bone .This application requires close surface contact to facilitate bone integration. In addition, the implant must have a porous coating or porous surface finish in order to promote long-term stability. This is the key reason why surfaces in cementless prostheses are coated with hydroxyapatite [3].

There have not been consolidated values for micromotion, few works have been carried out in this area owing to discrepancies in biological structure of human bones and tissues. The recommended values from different researchers are within the range of 0-200 µm. 200 µm is referred to as the osseointegration threshold. A study had also investigated theoretically, the interaction at the interface between the two materials in contact (Bone and SS316L) by applying Hertzian Contact Mechanics Model to ascertain the primary stability of the virtual Prosthetic joint [4]. An emerging focus on the investigation and analysis of the biomechanics of human bone is to generate preclinical information which is helpful for the researcher and orthopedicians has been seen. For this, a geometric model that acts like a natural bone has increasingly been considered to better understand the mechanics of the bone. Mechanical properties of bone are inhomogeneous which differentiate the bone geometrical structure as cortical and cancellous bone, in the same way as man-made engineering materials [4].

Bone as an elastic material

The bone tissue is a viscous-elastic material whose mechanical properties are affected by its deformation grade. The flexibility properties of the bone are provided by the collagen material of the bone. The collagen content gives the bone the ability to support tense loads. The bone is also a fragile material and its force depends on the load mechanism. The fragility grade of the bone depends on the mineral constituents that give it the ability to support compressive loads [5-7].

The bone is also viscoelastic, which means that it responds differently depending on the speed to which the load is applied and the length of the load. Bone has elastic response**;** when the load is firstly applied, a bone is deformed by a change in the extent or angular format. The bone is deformed up to 3% [8]. This is considered the elastic amplitude of the load-deformation curve because, when the load is removed, the bone is recovered and goes back to the original format or extent. With the continuous placement of load on the bone tissue, its deformation point is reached, after which the external fibers of the bone tissue will start to cede, experiencing micro-breaks and disconnection of the material within the bone.

A hard material will respond with a minimum deformation to the load increase. When the material fails in the end of the elastic phase, it is considered a fragile material. The glass is an example of fragile material. The bone is not so hard as the glass or metal, and, differently of those materials, it does not respond linearly, because it cedes and deforms not uniformly during the load placement phase. The higher the load imposed to the bone, the higher the deformation. In addition, if the load exceeds the elastic limits of the material, there will be a permanent deformation and failure of the material. If a material continues to over-elongate and over-deform in the plastic phase, it is known as flexible material. The skin is an example of material that is deformed considerably before the failure. The bone is a material that has properties that respond in both the fragile and the flexible mode as in figure 1 [8].

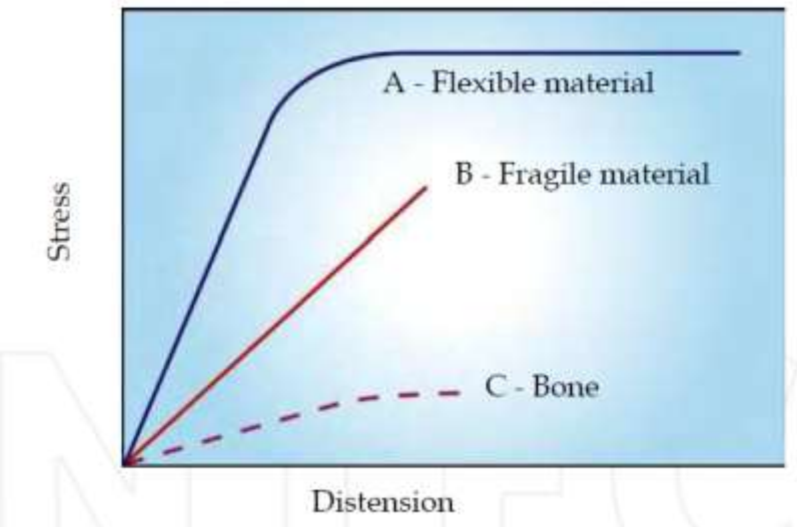

Fig-1: Stress Distention curve

The stress-distension curves illustrate the differences of behavior among (A) flexible material, (B) fragile material and (C) bone that has both fragile and flexible properties. When the load is applied, a fragile material responds linearly and fails or breaks before experiences any permanent deformation. The flexible material will get into the plastic area and will be considerably deformed before the failure or break. The bone is slightly deformed before the failure.

Bankoff [9] noted that the bone is considered viscoelastic because it responds differently when it receives loads in different speeds. When it receives the load quickly, the bone responds more rigidly, and may handle a higher load before it breaks and when it receives the load slowly, the bone is not so rigid or strong, breaking under lesser loads. The bone tissue starts to deform permanently and eventually breaks if the load continues in the non-elastic phase. Thus, when the load is removed, the bone tissue does not retake the original extent and is permanently elongated. [9-11].

MATHEMATICAL MODEL

Considering figures 2 and 3, it could be noted that as a result of the applied force F_A the implant of mass m moves up and down. The bone and the motion are resisted by the upward force F_B from the bone and shear force F_S at bone/implant interface. If as a result of the applied force, the implant moves at a micro-velocity v, then the equation of motion from Newton's law of motion becomes.

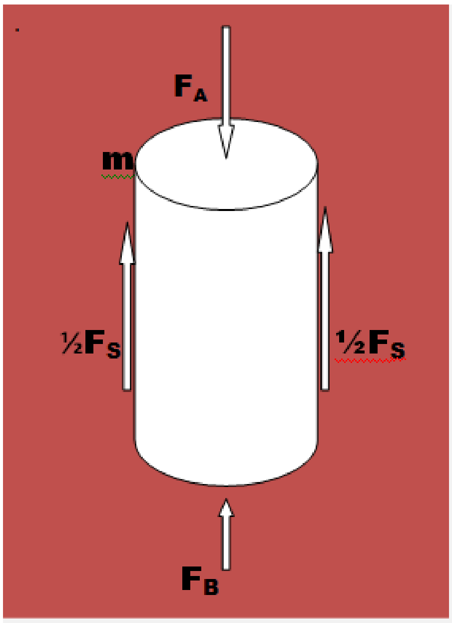

Fig-2: Implant in bone system

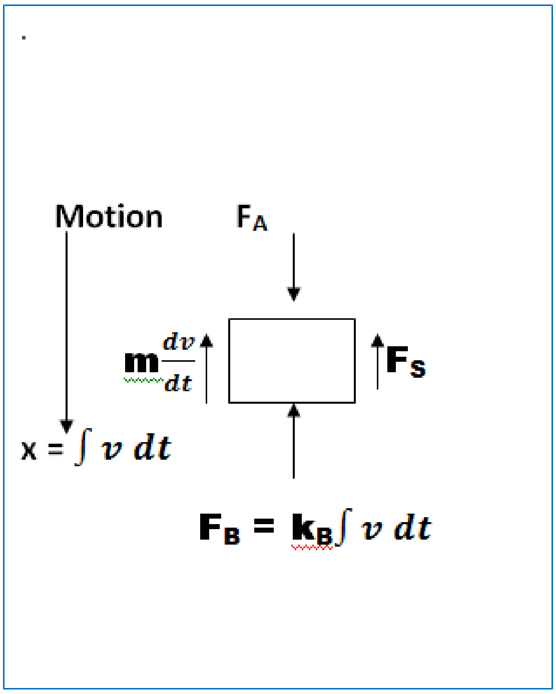

Fig-3: Motion analysis of implant in bone

$$m\frac{dv}{dt} = F_A - F_B - F_S; \quad (1)$$

F_S and F_A can be quantified using spring analogue and equation of motion solved.

Note: $\delta = x = \int v\, dt$ (2)

Equation 2 represents the micro displacement as a result of the micro velocity.

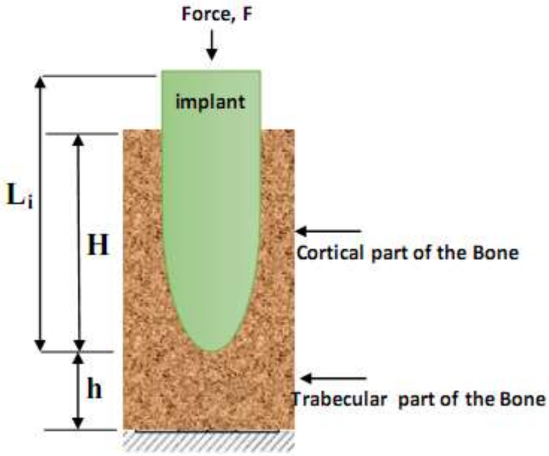

Fig-4: Implant in bone system

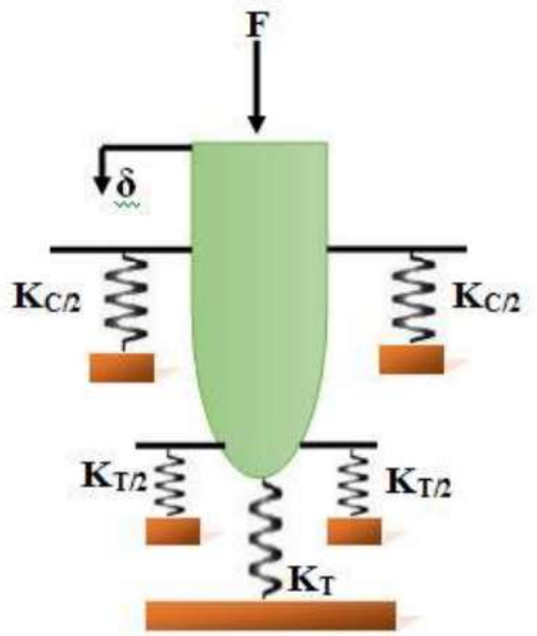

Fig-5: Spring representation of bone surface

Figure 4 shows the mechanical description of relative motion occurring at the bone-interface when related to elastic system using spring models. The implant rests on the layer of the elastic surface and the implant is elastically supported by the surrounding cortical and trabecular bone. Here, the displacement of the implant equals the micromotion depending on the stiffness of both cortical and trabecular (cancellous) bones. When the implant is axially loaded, due to the elastic modulus of the bone when compared to that of the implant, (stainless steel), the deformation of the implant is neglected. That is, $E_i >> (E_C, E_T)$ where E_i ,E_C and E_T are the elastic moduli of the implant, cortical part of the bone and the trabecular part of the bone respectively.

K_T, K_C and K_i are the stiffness of the trabacular bone, cortical bone and the implant respectively.

Li = length of the implant, h = height of the bone underneath the implant and, H = the reflecting height of the cortical part of the bone around the implant.

The elastic supports at different regions in figure 3a were replaced with spring systems. From the relation,

Young modulus = stress/strain (3)

From figure b, the $K_{C/2}$ are in parallel to each other, therefore the equivalent stiffness of the cortical part equals,

$K_{Teq} = K_T/4$ (4)

$K_{Ceq} = \frac{K_C}{4}$ (5)

Similarly,
$K_{Teq1} = \frac{K_{T/4}}{4}$ (6)

The total equivalent stiffness;
$K_{eq} = \frac{K_T}{4} + \frac{K_C}{4} + K_T = \frac{1}{4}(K_C + 5K_T)$ (7)

From Hooke's law;
Δ = FL/AE (8)

The equivalent arrangement of the spring is as shown in fig 3 below;

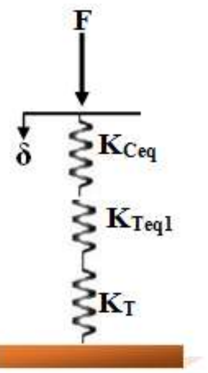

Fig-6: Equivalent spring arrangement for the cortical and trabecular bones

A= cross sectional area, L is the original length and E is the young modulus of an elastic body and Δ is the displacement. In this context, **Δ is δ.**

$\delta = \frac{FL_i}{AE_i} + \frac{Fh}{AE_T} + \frac{FH}{AE_C}$ (9)

$\delta = \frac{1}{\frac{AE_i}{L}} + \frac{1}{\frac{AE_T}{h}} + \frac{1}{\frac{AE_C}{H}}$ (10)

F = K δ (11)

This then implies that,
δ= F/K (12)

Relating equations 12 to 8,
K = AE/L (13)

Therefore,
$\delta = F(\frac{1}{K_i} + \frac{1}{K_{Teq}} + \frac{1}{K_{Ceq}})$ (14)

Also, equation 14 can be related to equation 2 as;

$$\int v\,dt = F\left(\frac{1}{K_i} + \frac{1}{K_{Teq}} + \frac{1}{K_{Ceq}}\right) \quad (15)$$

Equation 15 relates the velocity at which the implant moves down in the bone at the implant- bone surface at a specific time, the axial force on the head of the implant and the stiffnesses (elastic constants) of the implants and the bones.

CONCLUSION

This paper shows the importance of mathematical relationship in area of biomechanics study and brought brief discussions the interaction between the implant and the host bone at elastic interface which results into micromotion as a result of the axial force applied on the implant. The mathematical model so derived expresses the relation among the velocity at which the implant moves down in the bone at the implant-bone surface at a specific time. The axial force on the head of the implant and the stifnesses (elastic constants) of the implant and the bones. More work should be done in this area to ascertain the validity of the mathematical model in determining micromotion at implant bone interface.

REFERENCES

1. Hürkamp A, Lutz A, Nackenhorst U. Numerical investigations on the fixation of cemented and uncemented endoprostheses. PAMM. 2011 Dec 1;11(1):89-90.
2. Lutz A, Nackenhorst U. A Computational Approach on the Osseointegration of Bone Implants Based on a Bio-Active Interface Theory. GAMM-Mitteilungen. 2009 Dec 1;32(2):178-92.
3. Katti KS, Verma D. Materials for Joint Replacement: In Joint Replacement Technology, P. Revell (Ed.), Cambridge, Woodhead Publishing Ltd. 2008.
4. Gabriel Oladeji Bolarinwa, Sam Omenyi. Verification of the Primary Stability of Bone-Implant Non Manifold Assembly (Prosthetic Joint) using Hertzian Contact Mechanics Model, Sch. J. Eng. Tech., 2016; 4(8) 370-374.
5. Alberts B. Molecular Biology of the Cell, 3rd Ed., Garland Press. 1994.
6. Junqueira LC, Carneiro J. Biologia Celular e Molecular, 6th Ed., Rio de Janeiro, Brasil. 1997.
7. Junqueira LC, Carneiro J. Histologia Básica, 9th Ed., Guanabara Koogan, Rio de Janeiro, Brasil. 1999.
8. Choi K, Goldstein SA. A Comparison of the Fatigue Behavior of Human Trabeculae and Cortical Bone Tissue, Journal of Biomechanics. 1992;25:1371.
9. Bankoff ADP. Morfologia e Cinesiologia Aplicada ao Movimento Humano, Editora Guanabara Koogan, Rio de Janeiro, Brasil. 2007.
10. Hay ED. (Editor). Cell Biology of Extracellular Matrix, Plenum. 1982.
11. Holtrop ME. The Ultra Structure of Bone, Ann Clin Lab Sci. 1975;5:264.
12. Perez MA, Fornell P, Garcia-Aznar JM, Doblaret M. Validation of bone remodelling models applied to different bone types using Mimics. Available online at: www. materialise. com/ download. 2007.