Scholars Journal of Engineering and Technology (SJET) **ISSN 2321-435X (Online)**
Sch. J. Eng. Tech., 2016; 4(5):215-219 **ISSN 2347-9523 (Print)**
©Scholars Academic and Scientific Publisher
(An International Publisher for Academic and Scientific Resources)
www.saspublisher.com

Research Article

Aerodynamics of Spinning Sphere in Ideal Flow

R. C. Mehta
Department of Aeronautical Engineering, Noorul Islam University, Kumaracoil 629180, India

***Corresponding author**
R. C. Mehta
Email: drrakhab.mehta@gmail.com

Abstract: Aerodynamic analysis is carried out using superimposition of uniform flow with doublet over a spinning sphere. Analytical solutions of the pressure distribution and lift force over the spinning sphere are obtained based on Kutta-Joukowski theorem with the assumption that flow field around the spinning sphere may not influence the synthesized flow. The theoretical aerodynamic analysis reveals that the lift force over the spinning sphere is directly proportional to circulation, which coincides with the experimental results of Bearman and Harvey. It is found that the lift force over the spinning sphere is less than the spinning circular cylinder. The tangential velocity component on the surface of the spinning sphere yields a cubic equation, which is having one real and two conjugate imaginary roots.
Keywords: Kutta-Joukowski theorem, Aerodynamics, Ideal Fluid, Sphere.

INTRODUCTION

Air flow around a spinning sphere produces a normal force perpendicular to the ball and is having significant interest in many sports [1] such as golf, baseball, tennis, table tennis, soccer, volleyball and cricket ball, because the balls used in these sports are having translate and rotate motion simultaneously.

Newton [2] noted that a spinning ball deviates in flight and explained that the surrounding air was responsible for the deviation of the flight path. Robins [3] has shown the deflection of musket balls in terms of their spin. Magnus [4] demonstrated that a rotating cylinder experienced a sideways force when mounted perpendicularly to a flow of air. The expression of the Magnus effect has credited from Lord Rayleigh [5]. Earlier explanation for the Magnus effect was based on Bernoulli's theorem [6] corresponding to invscid and incompressible flow. The lift force acting is caused by a pressure differential between two sides of the sphere, resulting from the velocity difference due to the rotation. Later on discovery of boundary layer due to viscous flow by Prandtl provides another explanation of the Magnus effect attributed asymmetric and flow separation [7].

Maccoll [8] was first quantitatively measured the Magnus force on a smooth spinning sphere of 6 inch diameter in an air stream. Davies [9] has calculated the lift and drag coefficients from the drift of golf ball at translation velocity 32 m/s and rotation speed less than 5000 rpm. Brigg [10] has carried out experiment in 6 ft wind tunnel using smooth ball of 3 inch diameter at spin rates up to 1800 rpm at freestream velocity of 125 ft/s at Reynolds number 2.4×10^5. Pressure at the equatorial surface over the spinning sphere was measured by Briggs and found that the resultant pressures are consistently in accord with the Magnus effect. Briggs has calculated using the experimental data that the lateral deflection was proportional to ωV_∞^2 . Bearman and Harvey [11] have shown that the lift on a rotating sphere is directly proportional to ωV_∞. Watts and Ferror [12] have found in the analysis of the aerodynamics of curve ball that the normal force over the spinning sphere is consistent with the Kutta-Joukowaski theorem [6, 13] that can be related a net circulation of an indeal flow over a two-dimensional object results in a lift force proportional to the product of the freestream velocity and circulation.

Poon *et al.* [14] have carried out numerical simulation of viscous flow over a stationary and rotating sphere using Fourier-Chebyshev spectral collocation. Ou *et al.* [15] have investigated unsteady fluid dynamics of viscous flow over a spinning smooth sphere using high order numerical method. Numerical tests have been performed to validate the solver for spinning and non-spinning spheres at laminar Reynolds number. Numerical simulations of viscous flow over soccer balls have been carried out by Barber *et al.* [16] using FLUENT software to analyze the aerodynamic coefficients. Numerical simulations used to solve spinning sphere but does not exhibits generalized relation between lift to spinning rate.

The aim of the present paper is to derive an analytical solution for steady, inviscid, incompressible and axisymmetric flow past a spinning sphere. The analysis is based on the assumption that the flow field may not affect due to spinning of the sphere. The analysis includes superimpose of uniform flow with double that synthesis of lifting flow over spinning sphere.

ANALYSIS

Figure 1 shows *r*, *φ*, *θ* spherical polar coordinates. The freestream flow is along the *z*-axis and the sphere is constrained to rotate in between the *y*- and *z*- axis. The sphere is constrained to rotate at a constant angular velocity.

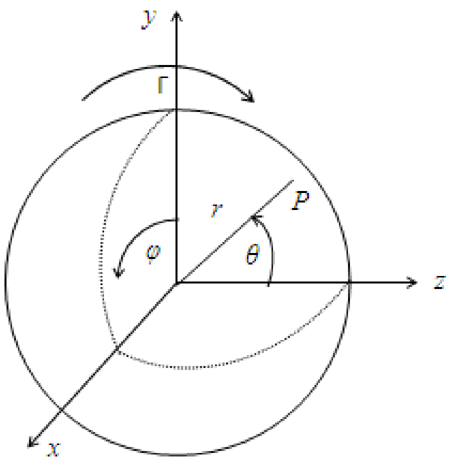

Fig-1: Spherical polar coordinate system

Figure 2 depicts systematic diagram represents superposition of uniform flow in the positive *z*-direction and with doublet. The doublet is so oriented that the source is placed upstream.

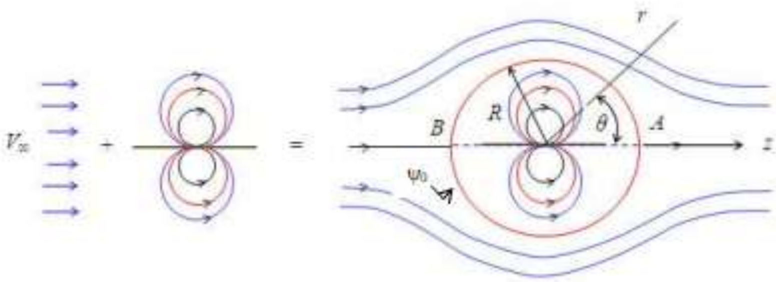

Fig-2: Superposition of a uniform flow and a doublet

It is important to mention here that source, sink and doublet represent three dimensional in nature of fluid mechanics. Let us consider for a uniform flow and a doublet combination the constant streamline ($\psi = \psi_0$) yields the following equation [13] as

$$\left(r^3 - \frac{\mu}{2\pi V_\infty}\right) sin^2\,\theta = 0 \qquad (1)$$

where – μ is the doublet strength with source placed upstream. Equation (1) can be satisfied if either sinθ = 0 or the quantity inside the parenthesis is zero, that is, when

$$\theta = 0 \quad \text{or} \quad \pi \qquad (2a)$$

and

$$r = \left(\frac{\mu}{2\pi V_\infty}\right)^{1/3} \qquad (2b)$$

But the value of *r* at points *A* and *B* as depicted in Fig. 2 on the *z*-axis is the radius of the sphere *R* as following

$$R = \left(\frac{\mu}{2\pi V_\infty}\right)^{1/3} \quad (3)$$

Thus, ψ_0 represents the streamline for a sphere of radius R and the z-axis. The sphere of radius R may be taken as a solid boundary as there is no flow across a streamline. Figure 2 shows two stagnation points on z-axis. They are $\left[\left(\frac{\mu}{2\pi V_\infty}\right)^{1/3}, 0\right]$ and $\left[\left(\frac{\mu}{2\pi V_\infty}\right)^{1/3}, \pi\right]$.

The sphere of radius R may be considered as a solid boundary as there is no flow across the stream line. For $r > R$, equations for stream function and velocity potential can be written as

$$\psi = \frac{V_\infty}{2}\left(r^2 - \frac{R^3}{r}\right) sin^2\,\theta \quad (4a)$$

$$\phi = V_\infty r\left[\frac{1}{2}\left(\frac{R}{r}\right)^3 + 1\right] cos\,\theta \quad (4b)$$

The velocity components in the radial and tangential directions are

$$v_r = V_\infty\left(1 - \frac{R^3}{r^3}\right) cos\,\theta \quad (5a)$$

$$v_\theta = -V_\infty\left(1 + \frac{R^3}{2r^3}\right) sin\,\theta \quad (5b)$$

The velocity components v_r and v_θ at the surface of a sphere, $r = R$, are 0 and $-(3/2)V_\infty\, sin\,\theta$, respectively. The stagnation points occur at $\theta = 0$ and π and the maximum velocities occur at $\theta = \pi/2$ and $\theta = (3/2)\pi$ with a value $|v_\theta|_{max} = -(3/2)V_\infty$. The pressure at any point on the sphere can be calculated by employing Bernoulli's equation. The pressure coefficient on the non-spinning sphere is

$$Cp = 1 - \frac{9}{4} sin^2\,\theta \quad (6)$$

Figure 3 displays the pressure distribution over non spinning cylinder and sphere and experimental data over spinning sphere of Briggs [10]. Briggs carried out pressure measurement over a spinning sphere of radius 3 inch with a freestream velocity of 125 ft/s in a wind tunnel. The experimental pressure distribution is quality identical trend as non-spinning sphere but differ qualitatively due to viscous effect and spinning of the sphere. The absolute magnitude of the pressure coefficient on a sphere is less than that for a circular cylinder [6] which is due three-dimensional relieving effect.

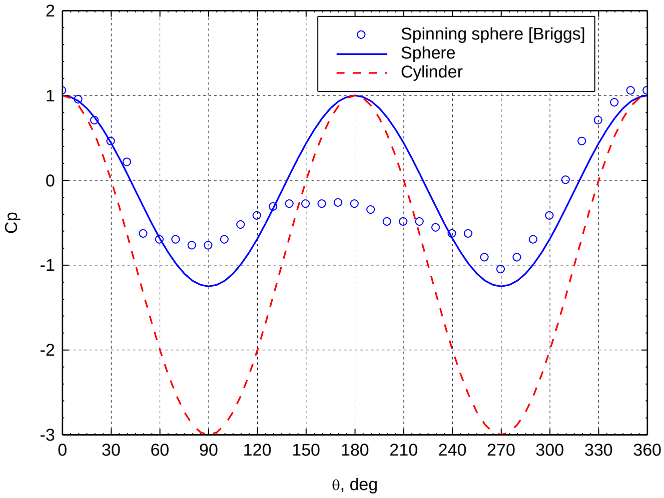

Fig-3: Pressure distribution over the non-spinning cylinder and sphere, and spinning sphere

We assume here that flow field around the spinning sphere will not affect synthesized flow. Figure 3 reveals that the profile of the pressure distribution over the spinning and non-spinning sphere remains same. Thus we superimpose the spinning sphere over the doublet with uniform velocity. The lift over the spinning sphere is obtained integrating over the equatorial plane of the spinning sphere can be written as

$$L = -\int_0^{2\pi}\left[\frac{\rho}{2}V_\infty^2 - \frac{\rho}{2}\left(\frac{3}{2}V_\infty \sin\varphi + \frac{\Gamma}{2\pi R}\right)^2\right]R\sin\theta d\theta \qquad (7)$$

The integration yields

$$L = \frac{3}{4}\rho_\infty V_\infty \Gamma \qquad (8)$$

The normal force over spinning cylinder per unit length [6] is $\rho_\infty V_\infty \Gamma$. It reveals that the normal force over the spinning sphere is less than the spinning circular cylinder.

$$V_\infty\left(1+\frac{R^3}{2r^3}\right)\sin\varphi + \frac{\Gamma}{2\pi r} = 0 \qquad (9)$$

$$2r^3 - \frac{r^2\Gamma}{\pi V_\infty} + R^3 = 0 \qquad (10)$$

The above cubic equation can be written as

$$\alpha^3 + v\alpha + w = 0 \qquad (11)$$

where

$$v = \frac{1}{3}\left(\frac{\Gamma}{2\pi V_\infty}\right)^2 \text{ and } w = \frac{2}{27}\left(\frac{\Gamma}{2\pi V_\infty}\right)^2 + \frac{1}{2}$$

Three roots [15] of equation (10) are x_1, x_2, and x_3 and written as

$$\alpha_1 = A_1 + B_1,\ \alpha_2, \alpha_3 = -\frac{1}{2}(A_1 + B_1) \pm \frac{\sqrt{3}}{2}(A_1 - B_1)i$$

where

$$A_1 = \left[-\frac{w}{2} + \sqrt{\left(\frac{w^2}{4} + \frac{v^3}{27}\right)}\right]^{1/3} \text{ and } B_1 = \left[-\frac{w}{2} - \sqrt{\left(\frac{w^2}{4} + \frac{v^3}{27}\right)}\right]^{1/3}$$

There is one real root and two conjugate complex root, when $\Gamma > 6\pi V_\infty\left(\frac{R^3}{2}\right)^{2/3}$

CONCLUSIONS

Theoretical aerodynamic is presented by superimposing of a uniform flow and doublet to compute the lift over a spinning sphere in ideal flow. Analytical is based on Kutta-Joukowski theorem with the assumption that flow field around the spinning sphere will not influence the synthesized flow. It is found that lift force over the spinning sphere is directly proportional to circulation which coincide experimental results of Bearman and Harvey, however, is less than spinning circular cylinder. The tangential velocity component on the surface of the spinning sphere yields a cubic equation which is having one real and two conjugate imaginary roots.

NOMENCLATURE

Cp = pressure coefficient
L = normal force
R = radius
V_∞ = freestream velocity
v_r, v_θ= velocity gradient in r and θ directions, respectively
r, φ, θ = spherical polar coordinates
x, y, z = Cartesian coordinates

α = root of the cubic equation
Γ = circulation
μ = doublet strength
ρ_∞= freestream density
ϕ = velocity potential
ψ = stream function
ω = rotation rate

REFERENCES

1. Mehta RD, Pallis JM; Sports Ball Aerodynamics: Effects of Velocity, Spin and Surface Roughness, in Materials and Science Sports, Edited by: EH. (Sam) Froes and S.J. Haake, TIMIS 184 Thorn Hill Road Warrendale, PA, USA, 2001; 185-197.
2. Newton I; New Theory about Light and Colors, Philosophical Transactions of the Royal society, 1671; 6:3078.
3. Robins B; New Principles of Gunnery, Hutton, London, 1742.
4. Magnus G; Ueber die abweichung der geschosse, und: Ueber eine auffallende erscheinung bei rotirenden korpern, Annalen der Physik, 164 (1), 1853, pp. 1–29.
5. Rayleigh L; On the Irregular Flight of a Tennis Ball, Messenger of Mathematics, Vol. 7(14), 1877, pp. 14-16.
6. Anderson JD; Fundamentals of Aerodynamics, International Student Edition, McGraw-Hill Book Company, Singapore, 1985.
7. Prandtl L; Essentials of Fluid Dynamics, Hafner Publishing Company, New York, 1952.
8. Maccoll JW; Aerodynamics of a Spinning Sphere, The Journal of the Royal Aeronautical Society, 1928; 32(213):777-798.
9. Davies JM; The Aerodynamics of Golf balls, American Journal of Physics, 1949; 20(9):821 -828.
10. Briggs LJ; Effect of Spin and Speed on the Lateral Deflection (Curve) of a baseball; and the Magnus effects for Smooth Spheres, American Journal of Physics, 1959; 27(8), 1959: 589 -596.
11. Bearman PW, Harvey J K; Golf Ball Aerodynamics, Aeronautical Quarterly, 1976; 21:112 - 122.
12. Watts RG, Ferrer R; The lateral force on a spinning sphere, American Journal of Physics, 1987; 55: 40–45.
13. Yuan SW; Foundations of Fluid Mechanics, Prentice-Hall of India Private Limited, New Delhi, India, 1969.
14. Poon KW, Ooi A, Giacobello M, Peralta C, Melatos A; Numerical Simulation of Flow Past a Stationary and Rotating Sphere, 16th Australasian Fluid Mechanics Conference, Crown Plaza, Gold Coast, Australia, 2007; 870-875.
15. Ou K, Castonguay P, Jameson A; Computational Sports Aerodynamics of a Moving Sphere: Simulating a Ping Pong Ball in Free Flight, 29th AIAA Applied Aerodynamics Conference, Honolulu, Hawaii, AIAA, 2011; 2011-3668.
16. Barber S, Chin SB, Carré MJ; Sports ball aerodynamics: A numerical study of the erratic motion of soccer balls, Computers and Fluids, 2009; 38(6): 1091-1100.
17. Grewal BS; Higher Engineering Mathematics, Khanna Publishers, 43rd Edition, Delhi, India, 2014; 49 – 50.