Research Article

Weighted Vertex PI Index for Some Special Graphs
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Abstract: The Padmakar-Ivan (PI) index is a Wiener-Szeged-like topological index which reflects certain structural features of organic molecules. Each structural feature of such organic molecule can be expressed as a graph. In this paper, we study the weighted vertex PI indices for some special graphs, such as $I_r(F_n)$, $I_r(W_n)$, $\sim_n F_n$, $\sim_n W_n$, $I_r(\sim_n F_n)$ and $I_r(\sim_n W_n)$.

Keywords: weighted vertex, PI indices, fan graph, wheel graph, gear fan graph, gear wheel graph, r-corona graph.

Introduction
The studies of topological indices for molecular structures have been conducted for over 35 years. Distance-based topological indices are numerical parameters of molecular structure, and play important roles in physics, chemistry and pharmacology science.

Specifically, let G be a molecular graph, then a topological index can be regarded as a positive real function $f: G \to \mathbb{R}^+$. As numerical descriptors of the molecular structure deduced from the corresponding molecular graph, topological indices have found several applications in theoretical chemistry, like QSPR/QSAR study. For instance, harmonic index, Wiener index, PI index, Randic index and sum connectivity index are introduced to reflect certain structural features and chemical characteristics of organic molecules. Recently, several articles contributed to reporting certain distance-based indices of special molecular graph (See Yan et al., [1-2], Gao et al., [3-4], Gao and Shi [5], Gao and Wang [6], Xi and Gao [7-8], Xi et al., [9], Gao et al., [10] for more detail). The notation and terminology used but undefined in this paper can be found in [11].

In this paper, we study the weighted vertex PI index of several simple connected graphs. Let $e=uv$ be an edge of the molecular graph G. The number of vertices of G whose distance to the vertex u is smaller than the distance to the vertex v is denoted by $n_u(e)$. Analogously, $n_v(e)$ is the number of vertices of G whose distance to the vertex v is smaller than the distance to the vertex u. Note that vertices equidistant to u and v are not counted. The weighted vertex PI index of a graph G was defined by Ilic and Milosavljevic [12] which is stated as follows:

$$PI_w(G) = \sum_{e=uv} (d(u) + d(v))[n_u(e) + n_v(e)].$$

In this paper, we determine the weighted vertex PI index for some special graphs. The organization of rest paper is as follows. First, we give some necessary definition in the next section. Then, the main result in this article is given in the third section.

Preliminaries
Definition 1. The graph $F_n=\{v\} \vee P_n$ is called a fan graph and the graph $W_n=\{v\} \vee C_n$ is called a wheel graph, where $P_n$ is a path with $n$ vertices and $C_n$ is a cycle with $n$ vertices.

Definition 2. Graph $I_r(G)$ is called r- crown graph of G which splicing r hang edges for every vertex in G. The vertex set of hang edges that splicing of vertex v is called r-hang vertices, note v*.
**Definition 3.** By adding one vertex in every two adjacent vertices of the fan path $P_n$ of fan graph $F_n$, the resulting graph is a subdivision graph called gear fan graph, denote as $\tilde{F}_n$.

**Definition 4.** By adding one vertex in every two adjacent vertices of the wheel cycle $C_n$ of wheel graph $W_n$, The resulting graph is a subdivision graph, called gear wheel graph, denoted as $\tilde{W}_n$.

**Main results and Proof**

**Theorem 1.** $PL_w(I_w(F_n)) = r^3(n^2 + 2n + 1) + r^2(8n^2 + 12n - 8) + r(n^3 + 9n^2 + 31n - 45) + (n^3 + 2n^2 + 21n - 36)$.

**Proof.** Let $P_n = v_1, v_2, \ldots, v_n$ and the $r$ hanging vertices of $v_i$ be $v_i^1, v_i^2, \ldots, v_i^r$ ($1 \leq i \leq n$). Let $v$ be a vertex in $F_n$ beside $P_n$, and the $r$ hanging vertices of $v$ be $v^1, v^2, \ldots, v^r$. Using the definition of weighted vertex PI index, we have

$$
PL_w(I_w(F_n)) = \sum_{i=1}^{n} (d(v) + d(v^i))(v_i(v^i) + v_i(v^i)) + \sum_{i=1}^{n} (d(v) + d(v^i))(v_i(v^i) + v_i(v^i)) + \sum_{i=1}^{n} (d(v) + d(v^i))(v_i(v^i) + v_i(v^i))
$$

$$
= (n + r + 1)(r + 1) + (2n + 1)(n + 2r + 2) + r(n - 1)(n + 1)(n + 2r + 3) + 2(n + 3)(2r + 5) + (n - 3)(2r + 6)(n + 1)(n + r + 1)
$$

$$
= r^3(n^2 + 2n + 1) + r^2(8n^2 + 12n - 8) + r(n^3 + 9n^2 + 31n - 45) + (n^3 + 2n^2 + 21n - 36).
$$

**Corollary 1.** $PL_w(F_n) = n^3 + 2n^2 + 21n - 36$.

**Theorem 2.** $PL_w(I_w(W_n)) = r^3(n^2 + 2n + 1) + r^2(8n^2 + 14n + 2) + r(n^3 + 8n^2 + 33n + 1) + (n^3 + 2n^2 + 29n)$.

**Proof.** Let $C_n = v_1, v_2, \ldots, v_n$ and $v_i^1, v_i^2, \ldots, v_i^r$ be the $r$ hanging vertices of $v_i$ ($1 \leq i \leq n$). Let $v$ be a vertex in $W_n$ beside $C_n$, and $v^1, v^2, \ldots, v^r$ be the $r$ hanging vertices of $v$. We denote $v_n, v_{n+1} = v_n, v_1$. In view of the definition of weighted vertex PI index, we infer

$$
PL_w(I_w(W_n)) = \sum_{i=1}^{n} (d(v) + d(v^i))(v_i(v^i) + v_i(v^i)) + \sum_{i=1}^{n} (d(v) + d(v^i))(v_i(v^i) + v_i(v^i)) + \sum_{i=1}^{n} (d(v) + d(v^i))(v_i(v^i) + v_i(v^i))
$$

$$
= (n + 1)(n + r + 1) + n(n + 1)(n + 2r + 3) + n(4r + 4)(2r + 6) + n(4r + 4)(2r + 6) + n(r + 1)(r + 1)
$$

$$
= r^3(n^2 + 2n + 1) + r^2(8n^2 + 14n + 2) + r(n^3 + 8n^2 + 33n + 1) + (n^3 + 2n^2 + 29n).
$$

**Corollary 2.** $PL_w(W_n) = n^3 + 2n^2 + 29n$.

**Theorem 3.** $PL_w(I_w(\tilde{F}_n)) = 4n^2r^3 + r^2(48n^2 - 32n) + r(2n^3 + 54n^2 - 48n) + (2n^3 + 26n^2 - 32n)$.

**Proof.** Let $P_n = v_1, v_2, \ldots, v_n$ and $v_{i+1}$ be the adding vertex between $v_i$ and $v_{i+1}$. Let $v_i^1, v_i^2, \ldots, v_i^r$ be the $r$ hanging vertices of $v_i$ ($1 \leq i \leq n$). Let $v_i^1, v_i^2, \ldots, v_i^r$ be the $r$ hanging vertices of $v_{i+1}$ ($1 \leq i \leq n-1$). Let $v$ be a vertex in $F_n$ beside $P_n$, and the $r$ hanging vertices of $v$ be $v^1, v^2, \ldots, v^r$. 

680
By virtue of the definition of weighted vertex PI index, we yield

$$P_{I_w}(I_1(F_n^q)) = \sum_{i=1}^{r} (d(v) + d(v'))(n_{ij}(vv') + n_{ij}(vv')) + \sum_{i=1}^{n} (d(v) + d(v'))(n_{ij}(vv') + n_{ij}(vv')) +$$

$$\sum_{i=1}^{n} \sum_{j=1}^{r} (d(v_i) + d(v_j))(n_{ij}(v_iv_j') + n_{ij}(v_jv_i')) + \sum_{i=1}^{n} (d(v_i) + d(v_j))(n_{ij}(v_iv_j') + n_{ij}(v_jv_i')) +$$

$$\sum_{i=1}^{n} (d(v_{i+1}) + d(v_{i+1}))(n_{ij}(v_{i+1}v_{i+1}) + n_{ij}(v_{i+1}v_{i+1})) +$$

$$+ \sum_{i=1}^{n} \sum_{j=1}^{r} (d(v_i) + d(v_j))(n_{ij}(v_i,v_{i+1}v_{i+1}) + n_{ij}(v_i,v_{i+1}v_{i+1}))$$

$$= r(2n+1)(n+1) + 2 \times 2n+1(n+2n) + (n-2)2n+1(n+2n+3) +$$

$$2 \times 2n+1(n+2n+3) + (n-2)2n+1(n+2n+3) + 2 \times 2n+1(n+2n+3) + (n-2)2n+1(n+2n+3) +$$

$$= 4n^3 + r^3 + (28n^2 + 18n + 2) + r(2n^3 + 52n^2 + 27n + 1) +$$

$$+ (2n^3 + 20n^2 + 10n).$$

**Corollary 3.** $P_{I_w}(I_1(F_n^q)) = 2n^3 + 26n^2 - 32n.$

**Theorem 4.** $P_{I_w}(I_1(W_n^q)) = (4n^2 + 6n+1)r^3 + r^3(28n^2 + 18n + 2) + r(2n^3 + 52n^2 + 27n + 1) +$$

$$+ (2n^3 + 20n^2 + 10n).$$

**Proof.** Let $C_i=v_1,v_2,...,v_n,$ and $v$ be a vertex in $W_n,$ beside $C_n,$ and $v_{i+1} \square$ be the adding vertex between $v_i$ and $v_{i+1}.$ Let $v^1,$ $v^2,$ ..., $v^r$ be the $r$ hanging vertices of $v$ and $v^1_i, v^2_i, ..., v^r_i$ be the $r$ hanging vertices of $v_i (1 \leq i \leq n).$ Let $v_{i+1} = v_{i+1},$ and $v_{i+1}, v'_{i+1}, ..., v'_{r+1}$ be the $r$ hanging vertices of $v_{i+1} (1 \leq i \leq n).$ Let $v_{i+1} = v_{i+1}, v_{i+1} = v_{i+1}.$ In view of the definition of weighted vertex PI index, we deduce

$$P_{I_w}(I_1(W_n^q)) = \sum_{i=1}^{r} (d(v) + d(v'))(n_{ij}(vv') + n_{ij}(vv')) + \sum_{i=1}^{n} (d(v) + d(v'))(n_{ij}(vv') + n_{ij}(vv')) +$$

$$\sum_{i=1}^{n} \sum_{j=1}^{r} (d(v_i) + d(v_j))(n_{ij}(v_iv_j') + n_{ij}(v_jv_i')) + \sum_{i=1}^{n} (d(v_i) + d(v_j))(n_{ij}(v_iv_j') + n_{ij}(v_jv_i')) +$$

$$\sum_{i=1}^{n} (d(v_{i+1}) + d(v_{i+1}))(n_{ij}(v_{i+1}v_{i+1}) + n_{ij}(v_{i+1}v_{i+1})) +$$

$$+ \sum_{i=1}^{n} \sum_{j=1}^{r} (d(v_i) + d(v_j))(n_{ij}(v_i,v_{i+1}v_{i+1}) + n_{ij}(v_i,v_{i+1}v_{i+1}))$$

$$= r(2n+1)(n+1) + n(2n+1)(r+1)(n+2n+3) + nr(2n+1)(r+1)(r+4) + n(2n+1)(r+1)(r+5) +$$

$$n(2n+1)(r+1)(r+5) + nr(2n+1)(r+1)(r+3) +$$

$$= (4n^2 + 6n+1)r^3 + r^3(28n^2 + 18n + 2) + r(2n^3 + 52n^2 + 27n + 1) +$$

$$+ (2n^3 + 20n^2 + 10n).$$

**Corollary 4.** $P_{I_w}(W_n^q) = 2n^3 + 20n^2 + 10n.$

**Conclusion**

Fan graph, wheel graph, gear fan graph, gear wheel graph and their $r$-corona graph are common structural features of organic molecules. The contributions of our paper are determining the weighted vertex PI index of these special structural features of organic molecules.
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