Scholars Journal of Engineering and Technology (SJET)
Sch. J. Eng. Tech., 2015; 3(6):609-611
ISSN 2321-435X (Online)
ISSN 2347-9523 (Print)

Research Article

Construction of single traveling wave solutions to modified YTSF equation

Hua xin
Northeast Petroleum University, Daqing City of Heilongjiang Province-163318, China.

***Corresponding author**
Hua xin
Email: xinhuayatou@126.com

Abstract: By the complete discrimination system for polynomial method, we obtained the classification of single traveling wave solutions to modified YTSF equation.
Keywords: complete discrimination system for polynomial method, modified YTSF equation.

INTRODUCTION

The classifications of single traveling wave solutions to some nonlinear differential equations have been obtained by the complete discrimination system for polynomial method proposed by Liu [1-3]. By this method a lot of nonlinear equation in mathematical physics has been solved [4].In the paper we study the modified YTSF equation [5].By complete discrimination system for polynomial method, we give the classification of its single traveling wave solutions.

Construction of solution

The modified YTSF equation reads as.

$$a^3cu^{(3)}+3a^2c\left(u'\right)^2+\left(3b^2+4a\omega\right)u'=0. \qquad (1)$$

Set $u'=v$, we have

$$v^{(2)}+\frac{3}{a}v^2+\frac{3b^2+4a\omega}{a^3c}v=0, \qquad (2)$$

multiply the equality by u', we obtain

$$v^{(2)}v'+\frac{3}{a}v^2v'+\frac{3b^2+4a\omega}{a^3c}vv'=0, \qquad (3)$$

after transforming the expression (3), we have

$$\left[\left(\frac{1}{2}v'\right)^2\right]'+\frac{3}{a}\left(\frac{1}{3}v^3\right)'+\frac{3b^2+4a\omega}{a^3c}\left(\frac{1}{2}v^2\right)'=0, \qquad (4)$$

by integrating the expression (4), the equality can be expressed by

$$\left(\frac{1}{2}v'\right)^2+\frac{3}{a}\frac{1}{3}v^3+\frac{3b^2+4a\omega}{a^3c}\frac{1}{2}v^2+c=0, \qquad (5)$$

the simplification of equality (5) can be given by

$$\left(v'\right)^2=a_3v^3+a_2v^2+a_1v+a_0, \qquad (6)$$

where $a_3 = -\frac{4}{a}$, $a_2 = -\frac{6b^2 + 8a\omega}{a^3 c}$, $a_1 = 0$, $a_0 = -4c$, Let $w = (a_3)^{\frac{1}{3}} v$, $d = a_2 (a_3)^{-\frac{2}{3}}$,

$d_1 = a_1 (a_3)^{\frac{1}{3}}$, $d_0 = a_0$,then Eq.(6)becomes:

$$\pm (a_3)^{\frac{1}{3}} (\xi - \xi_0) = \int \frac{1}{\sqrt{w^3 + d_2 w^2 + d_1 w + d_0}} dw. \quad (7)$$

We give the classification of its single traveling wave solutions as follows.

Case 1

$\Delta = 0,\ D_1 < 0,$ than is, $-27\left(\frac{2d_2^3}{27} + d_0 - \frac{d_1 d_2}{3}\right)^2 - 4\left(d_1 - \frac{d_2^2}{3}\right)^3 = 0$ and $d_1 - \frac{d_2^2}{3} < 0.$ Then we have

$F(w) = (w-\alpha)^2 (w-\beta),\ \alpha \neq \beta.$ if $w > \beta,$ the solutions are given as follows.

$$v = (a_3)^{-\frac{1}{3}} \left[(\alpha - \beta) \tanh^2 \left(\frac{\sqrt{\alpha - \beta}}{2} (a_3)^{\frac{1}{3}} (\xi - \xi_0) \right) + \beta \right], \alpha > \beta, \quad (8)$$

$$v = (a_3)^{-\frac{1}{3}} \left[(\alpha - \beta) \coth^2 \left(\frac{\sqrt{\alpha - \beta}}{2} (a_3)^{\frac{1}{3}} (\xi - \xi_0) \right) + \beta \right], \alpha > \beta, \quad (9)$$

$$v = (a_3)^{-\frac{1}{3}} \left[(\alpha - \beta) \tan^2 \left(\frac{\sqrt{\alpha - \beta}}{2} (a_3)^{\frac{1}{3}} (\xi - \xi_0) \right) + \beta \right],\ \alpha < \beta. \quad (10)$$

Case 2

$\Delta = 0,\ D_1 = 0,$ than is, $-27\left(\frac{2d_2^3}{27} + d_0 - \frac{d_1 d_2}{3}\right)^2 - 4\left(d_1 - \frac{d_2^2}{3}\right)^3 = 0$ and $d_1 - \frac{d_2^2}{3} = 0.$ Then we have

$F(w) = (w-\alpha)^3.$ The solution is given by

$$v = 4(a_3)^{-\frac{2}{3}} (\xi - \xi_0)^{-2} + \alpha \quad (11)$$

Case 3

$\Delta > 0,\ D_1 < 0,$ than is, $-27\left(\frac{2d_2^3}{27} + d_0 - \frac{d_1 d_2}{3}\right)^2 - 4\left(d_1 - \frac{d_2^2}{3}\right)^3 > 0$ and $d_1 - \frac{d_2^2}{3} < 0.$ Then

$F(w) = (w-\alpha)(w-\beta)(w-\gamma).$ We suppose that $\alpha < \beta < \gamma.$ When $\alpha < w < \beta,$ we have

$$v = (a_3)^{-\frac{1}{3}} \left[\frac{\gamma - \beta sn^2 \left(\frac{\sqrt{\gamma - \alpha}}{2} (a_3)^{\frac{1}{3}} (\xi - \xi_0), m \right)}{cn^2 \left(\frac{\sqrt{\gamma - \alpha}}{2} (a_3)^{\frac{1}{3}} (\xi - \xi_0), m \right)} \right], \alpha > \beta, \quad (12)$$

where $m^2 = \frac{\beta - \alpha}{\gamma - \alpha}.$

Case 4

$\Delta < 0,$ than is, $-27\left(\frac{2d_2^3}{27}+d_0-\frac{d_1 d_2}{3}\right)^2-4\left(d_1-\frac{d_2^2}{3}\right)^3<0.$ Then we have

$F(w)=(w-\alpha)(w^2+pw+q),\ p^2-4q<0.$ Furthermore, we have

$$v=(a_3)^{-\frac{1}{3}}\left[\alpha-\sqrt{\alpha^2+p\alpha+q}+\frac{2\sqrt{\alpha^2+p\alpha+q}}{1+cn\left(\left(\alpha^2+p\alpha+q\right)^{\frac{1}{4}}(a_3)^{\frac{1}{3}}(\xi-\xi_0),m\right)}\right], \qquad (13)$$

where $m^2=\frac{1}{2}\left(1-\frac{\alpha+\frac{p}{2}}{\sqrt{\alpha^2+p\alpha+q}}\right).$

ACKNOWLEDGEMENT

Supported by 《The research about building applied statistics undergraduate talents training system》No:GBC1214006

REFERENCES

1. Cheng-Shi Liu; Exact traveling wave solutions for (1+1)-dimensional dispersive long wave eqution, Chin.Phy, 2005; 14(9): 1710-1715.
2. Cheng-Shi Liu; All Single Traveling Wave Solutions to (3+ 1)-Dimensional Nizhnok Novikov Veselov Equation. Commun. Theor. Phys (Beijing, China), 2006; 45: 991-992.
3. Cheng-Shi Liu; Representations and classification of traveling wave solutions to sinh-Gördon equation[J]. Communications in Theoretical Physics, 2008; 49(1): 153.
4. Cheng-Shi Liu; Applications of complete discrimination system for polynomial for Classifica-tions of traveling wave solutions to nonlinear differential equations. Computer Physics Communications, 2010; 181(2): 317-324.
5. Zhang S, Zhang HQA; transformed rational function method for (3+ 1)-dimensional potential Yu–Toda–Sasa–Fukuyama equation. Pramana, 2011, 76(4): 561-571.