Research Article

Optimizing horizontal wells length of low permeability reservoir by fuzzy math
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Abstract: M69 block of Jilin province is low permeability reservoir which the choice of horizontal wells length is one of the essential problems to the oilfield development. In this paper, fuzzy evaluation method will be applied to optimize the horizontal length of horizontal wells considering layer permeability, pressure difference, fracture quantity, fracture half length, fracture width and fracture interval six factors comprehensively. The example indicates that the calculation is accurate with promotional value.

Keywords: fuzzy math; horizontal wells; horizontal section length; low permeability

INTRODUCTION
Utilizing horizontal wells to develop fields is a significant technique that is widely used. Completion technique to the horizontal well, bore size, well track, horizontal section length and many other factors may impact the production of horizontal wells, yet confirming reasonable horizontal section length places the critical role in the development and design of horizontal wells[1]. While the length of the horizontal wells is not proportional to the production[2] due to the increasingly difficult operation, borehole wearing, oil pollution during the well drilling process and so on, therefore there exists a reasonable horizontal section length to the horizontal wells[3]. The factors affect horizontal section length include layer permeability, pressure difference and so forth which are fuzzy, uncertain and unfathomable. Based on these characteristics is difficult to establish optimized model of the horizontal section length, while fuzzy evaluation method can solve these multivariate and multi-objective fuzzy questions objectively[4]. According to M69 block of Jilin, utilizing fuzzy evaluation method to adjust actual parameters of the field, so can optimize a suitable horizontal section length to this block.

FUZZY OPTIMIZE MODEL
Evaluated indication
Combining reservoir characters of the M69 block of Jilin province, we select the following 6 factors as the evaluated indication to optimize horizontal section length: layer permeability, pressure difference, fracture quantity, fracture half length, fracture width and fracture interval.

Eigenvector matrix to the indexes
If there exist evaluation indication with the quantity m that makes up the evaluation indication set for programs with the quantity n, each evaluation indication can use eigenvectors to judge the programs[5]. And the eigenvector matrix to the indexes as follows:

\[
Y = \begin{bmatrix}
y_{11} & y_{12} & \cdots & y_{1n} \\
y_{21} & y_{22} & \cdots & y_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
y_{m1} & y_{m2} & \cdots & y_{mn}
\end{bmatrix} = y_{ij}
\]

\((1)\)

\(y_{ij} (i=1, 2, ..., m; j=1, 2, ..., n)\) is the eigenvector to the evaluated indication i of the program j.

Degree of membership matrix
According to analysis of each indication, if the increased value is beneficial to improve the production, we can use the following formula to describe the degree of membership:
Among them, \( r_{ij} \) is the degree of membership to the evaluated indication \( i \) of the program \( j \), and \( y_{\text{min}}, y_{\text{max}} \) is the minimum and maximum respectively to the evaluated indication \( i \) of the set[6].

So on the basis of the formula (2) and (3), eigenvector matrix to the indexes can be inverted into membership matrix:

\[
R = \begin{bmatrix}
    r_{11} & r_{12} & \cdots & r_{1n} \\
    r_{21} & r_{22} & \cdots & r_{2n} \\
    \vdots & \vdots & \ddots & \vdots \\
    r_{m1} & r_{m2} & \cdots & r_{mn}
\end{bmatrix} = r_{ij}
\]

(4)

We define excellent program \( G \) and inferior program \( S \) by the principle that membership of excellent program with the quantity of \( m \) is the maximum to the all programs.

\[
G = (g_1, g_2, \cdots, g_m) = (r_{11} \lor r_{12} \lor \cdots \lor r_{1n}, r_{21} \lor r_{22} \lor \cdots \lor r_{2n}, r_{m1} \lor r_{m2} \lor \cdots \lor r_{mn})
\]

(5)

\[
S = (s_1, s_2, \cdots, s_m) = (r_{11} \land r_{12} \land \cdots \land r_{1n}, r_{21} \land r_{22} \land \cdots \land r_{2n}, r_{m1} \land r_{m2} \land \cdots \land r_{mn})
\]

(6)

**DETERMINING THE WEIGHTS**

In order to hierarchy the question, we utilize analytical hierarchy process(AHP) and induct the ratio scaling method from 1 to 9. Then we get a judgment matrix of which implication is illustrated in table-1[7]:

<table>
<thead>
<tr>
<th>Scales</th>
<th>Implication</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Compared to 2 factors, equally important</td>
</tr>
<tr>
<td>2</td>
<td>Mid-value between adjacent 1 and 3</td>
</tr>
<tr>
<td>3</td>
<td>Compared to 2 factors, the former is slightly more important than the latter.</td>
</tr>
<tr>
<td>4</td>
<td>Mid-value between adjacent 3 and 5</td>
</tr>
<tr>
<td>5</td>
<td>Compared to 2 factors, the former is more important than the latter.</td>
</tr>
<tr>
<td>6</td>
<td>Mid-value between adjacent 5 and 7.</td>
</tr>
<tr>
<td>7</td>
<td>Compared to 2 factors, the former is strongly more important than the latter.</td>
</tr>
<tr>
<td>8</td>
<td>Mid-value between adjacent 7 and 9.</td>
</tr>
<tr>
<td>9</td>
<td>Compared to 2 factors, the former is extremely more important than the latter.</td>
</tr>
<tr>
<td>Reciprocal value</td>
<td>If the important ratio of element ( i ) and ( j ) is ( a_{ij} ), the important ratio of element ( j ) and ( i ) is ( a_{ji} = 1/a_{ij} ).</td>
</tr>
</tbody>
</table>

Defining \( B_{ij} = \log a_{ij} \)  \hspace{1cm} (7)

\[
D_{ij} = \frac{\sum_{k=1}^{N} (B_{ik} - B_{jk})}{N} \quad (8)
\]

\[
a_{ij}^* = 10^{D_{ij}} \quad (9)
\]

Judgment matrix \(a_{ij}^*\) and \(a_{ij}\) is completely equivalent meeting principle of consistency. Multiply the elements of each row and calculate the result by nth root, then define it as \(M_i\):

\[
P_i = \sqrt[n]{\prod_{j=1}^{m} a_{ij}^*} \quad (10)
\]

\[
P = (P_1, P_2, \ldots, P_n)^T\], and then standardize \(P_i\):

\[
W_i = P_i / \sum_{k=1}^{n} P_k \quad (11)
\]

At final, we can obtain the weight vector: \(W = (W_1, W_2, \ldots, W_n)^T\).

Next we compute membership of excellent program belonging to each program, namely the optimal membership to the program. Then determining the optimal sequence according to the principle, the maximum membership.

\[
u_j = \frac{1}{1 + \sum_{i=1}^{m} \left( \frac{|r_j - g_i|}{W_i} \right)^2} \quad (12)
\]

EXEMPLARY OF APPLICATION
Now we analyze the data of HP29, HP5, HP26, HP44 and HP24 from M69 block listing in table 2.

<table>
<thead>
<tr>
<th>Well name</th>
<th>length of horizontal section L(m)</th>
<th>layer permeability K(md)</th>
<th>pressure difference (\Delta P) (MPa)</th>
<th>fracture quantity M(Piece)</th>
<th>fracture half length D(m)</th>
<th>fracture width A(m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HP29</td>
<td>400</td>
<td>12.10</td>
<td>10.84</td>
<td>4</td>
<td>35.83</td>
<td>146.71</td>
</tr>
<tr>
<td>HP5</td>
<td>500</td>
<td>4.37</td>
<td>8.58</td>
<td>6</td>
<td>38.89</td>
<td>134.41</td>
</tr>
<tr>
<td>HP26</td>
<td>600</td>
<td>37.29</td>
<td>3.36</td>
<td>5</td>
<td>46.67</td>
<td>116.10</td>
</tr>
<tr>
<td>HP44</td>
<td>700</td>
<td>25.32</td>
<td>9.90</td>
<td>7</td>
<td>33.33</td>
<td>132.44</td>
</tr>
<tr>
<td>HP24</td>
<td>800</td>
<td>3.73</td>
<td>9.22</td>
<td>8</td>
<td>29.17</td>
<td>114.40</td>
</tr>
</tbody>
</table>

Vector matrix to the indexes
From formula (1),

\[
Y = \begin{bmatrix}
12.10 & 4.37 & 37.29 & 25.32 & 3.73 \\
10.84 & 8.58 & 3.36 & 9.90 & 9.22 \\
4 & 6 & 5 & 7 & 8 \\
35.83 & 38.89 & 46.67 & 33.33 & 29.17 \\
146.71 & 134.41 & 116.10 & 132.44 & 114.40 \\
0.0061 & 0.0056 & 0.0041 & 0.0061 & 0.0042
\end{bmatrix}
\]

Degree of membership matrix
For the factors impacting deliverability, the value of layer permeability, pressure difference, fracture quantity, fracture half length, fracture width is direct proportional to deliverability, while fracture interval is converse. Then we can compute the membership matrix \(R\) by formula (2), (3) and (4).

\[
R = \begin{bmatrix}
0.25 & 0.02 & 1 & 0.64 & 0 \\
1 & 0.70 & 0 & 0.87 & 0.78 \\
0 & 0.50 & 0.25 & 0.75 & 1 \\
0.62 & 0.44 & 0 & 0.76 & 1 \\
1 & 0.62 & 0.05 & 0.56 & 0 \\
1 & 0.75 & 0 & 1 & 0.05
\end{bmatrix}
\]

Calculate the value of G and B
From formula (5) and (6),
\[G = (1, 1, 1, 1, 1, 1), \quad B = (0, 0, 0, 0, 0, 0)\]

Determining the weights
Comparing to these indexes on the basis of the method AHP, we establish judgment matrix and calculate weights by using the formula from (7) to (11).

<table>
<thead>
<tr>
<th>Indexes</th>
<th>Layer permeability</th>
<th>Pressure difference</th>
<th>Fracture quantity</th>
<th>Fracture interval</th>
<th>Fracture half length</th>
<th>Fracture width</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weights</td>
<td>0.05</td>
<td>0.06</td>
<td>0.10</td>
<td>0.16</td>
<td>0.25</td>
<td>0.38</td>
</tr>
</tbody>
</table>

Therefore we can know that,
\[W = (W_1, W_2, \ldots, W_n)^T = (0.05, 0.06, 0.10, 0.16, 0.25, 0.38)^T\]

Calculate optimal membership to each program
Based on formula (12),
\[U = (0.595, 0.787, 0.014, 0.928, 0.164)^T\]

Due to the principle that is the maximum membership, we order these 5 programs from excellent lever to inferior level: M_4, M_2, M_1, M_5, M_3. Obviously the length 700m is the optimal horizontal section length for horizontal wells.

CONCLUSION
(1) For determining the horizontal section length of horizontal wells in M69 block, we consider the 6 factors: layer permeability, pressure difference, fracture quantity, fracture half length, fracture width and fracture interval.

(2) Combined the fuzzy math with analytical hierarchy process (AHP), there forms a fuzzy evaluation method. During the calculating process, we eliminate human subjectivity and avoid uniformity, so the result is reasonable and reliable.

(3) The final conclusion is consistent with selected program realistically, thence this method can be utilized to solve oil and gas engineering issues similarly.

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