**Research Article**

**Correction of Friction Resistance Coefficient Based on Filled Function Method**

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**Abstract:** The changed friction resistance coefficient of the pipeline after the oilfield water injection pipe network system is used for a long term cannot reflect the actual conditions of the pipeline accurately. As the main parameter in the water injection system, the friction resistance coefficient of the pipeline concerns to the accuracy of the simulating calculation of water injection system and the efficiency of operation optimization.

**Keywords:** Water injection pipeline, Friction coefficient, Filled function, Global optimization

**INTRODUCTION**

The research on whole optimization problem has become one of hot issues attracted great attention.

The filled function method is one of the better methods for jumping to the better minimum point from the current minimum point and applied in the correction of the friction resistance coefficient of the pipeline[1-6]. Excellent correction result is obtained based on the multi-operating data.

**METHODS**

**Correction and optimization mathematical model of pipeline friction resistance coefficient**

The friction resistance coefficient vector \( C \) is used as the decision variable, the quadratic sum of the difference of the pressure calculated value of each node of all working conditions and the actual value is used as the target function to set up an optimization model.

Assuming that the water injection pipeline network has \( m \) pipelines, the working conditions are \( L, n \) is nodes, and \( C_j \) is the friction resistance coefficient of \( j \) pipeline, \( H^l_i \) is the pressure of \( i \) node of \( l \) working condition, and \( C = (C_1, C_2, \cdots, C_m)^T \) is recorded as the friction resistance coefficient vector. The pressure vector of \( l \) working condition is \( H^l = (H^l_1, H^l_2, \cdots, H^l_n)^T \), and \( H^l_0 \) is the actual pressure vector of \( l \) working condition. If \( C \) and \( H^l_0 \) are known, any component of \( H^l \) is used as the pressure of the reference point to obtain the pressure \( H^l \) of the corresponding node. Under the precondition that each \( H^l_0 \) is known, \( H^l \) is the function of vector \( C \) and recorded as \( H^l = H^l(C) \). The target function should be set as

\[
f(C) = \sum_{l=1}^{L} \| H^l(C) - H^l_0 \|^2 \tag{1}\]

\( C_k \) of each pipeline has a scope of \( C_k^{\min} \leq C_k \leq C_k^{\max} \), and in this text, the lower limit of the friction resistance coefficient is 80 and the upper limit is 120.

So, the correction and optimization mathematical model of pipeline friction resistance coefficient in many working conditions is
min \( f(C) = \sum_{i=1}^{L} \| H'(C) - H_0^i \|^2 \)  
\[ s.t. \quad C_k^{\min} \leq C_k \leq \max C_k^{\max}, \quad k = 1, 2, \cdots, m \]

Theoretically, the optimization model has at least one \( C \) so that the target function value is 0, that is to say the calculated pressure is equal to the actual pressure value. \( C \) meeting the constraint conditions and enabling the target function value to be 0 is not always actual friction resistance coefficient expect that the optimization model has an unique solution. The more of the working conditions, the greater possibility of the unique solution of the model, and it has great possibility that the optimal solution is the real friction resistance coefficient. The model (2) has at least one solution in many working conditions, when the working condition value is the value, the optimization model (2) has the unique solution, evidence has not been give theoretically, and it is the problem to be researched in future. At present, the target of the correction friction resistance coefficient is to seek for \( C \) meeting the conditions, so that \( f(C) \) is minimum, and the \( C \) is the correction value of friction resistance coefficient.

Filled function selected by correction mathematical model of friction resistance coefficient

If function \( P(x, x^*) \) meets to the following conditions:

1. \( x^* \) is a local maximum point of \( P(x, x^*) \);
2. For any \( x \in S_1, \nabla P(x, x^*) \neq 0 \) and \( S_1 = \{ x \mid f(x) \geq f(x^*), x \in X \setminus \{x^*\} \} \);
3. If \( x^* \) is not the whole minimum point, \( P(x, x^*) \) exists local minimum points on collection \( S_2 \), in which \( S_2 = \{ x \mid f(x) < f(x^*), x \in X \} \).

Function \( P(x, x^*) \) is the filled function of \( f(x) \) at the local minimum point \( x^* \).

\( L(P) \) is recorded as the collection of the local minimum point of \( \{ \min f(x) \mid s.t. x \in \Omega \} \) and \( G(P) \) is the collection of whole minimum point.

The expression formula of one-parameter filled function is as follows:

\[
F(x, x^*, \rho) = -\|x - x^*\|^3 + \rho \left[ \min \{0, f(x) - f(x^*)\} \right]^3
\]

In which, \( x^* \) is the present local minimum point of virtual loop equation \( F(Q)_i - F(Q)_k = (\sum h)_i, F(Q) \) shows the flow and hydraulic characteristics of water source, and \( \rho > 0 \) is the parameter.

When \( \rho > 0 \) is great enough, \( F(x, x^*, \rho) \) is a filled function meeting the nature.

Correction mathematical model of friction resistance coefficient

Assuming that \( x^* \in L(P), x^i \) is the current iteration point and \( d^i \) is the current iteration direction. We have the following theorem: for given constant \( \lambda_L, \lambda_U \) meets \( 0 < \lambda_L < \lambda_U \), ordering \( x' \in X \) and \( x^i = x^i + d^i \in X \), \( d^i \) here is the searching direction at \( x^i \) so that \( \lambda_L \leq ||d^i|| \leq \lambda_U \), ordering \( \theta \) is the included angle of \( x^i - x^* \), the following formula is equivalent:

1. \( \|x^{i+1} - x^*\| > \|x^i - x^*\| \);  
2. \( 2(x^i - x^*)^T d^i + ||d^i||^2 > 0 \);
(3) \[ \cos \theta^i > -\frac{\|d^i\|}{2\|x^i - x^*\|}. \]

(4) \( (x^i - x^*)^T d^i + (x^{i+1} - x^*)^T d^i > 0 \)

Specifically, when \( (x^k - x^*)^T d^k \geq 0, \quad \forall k = 0, 1, \cdots, i-1 \), \( \|x^i - x^*\|^2 \geq \lambda_i^2 + \|x^0 - x^*\|^2 \) is set up.

If \( d \) meets \( (x^i - x^*)^T d \geq 0 \), the iteration point shall reach to the boundary of \( X \) when the iteration step is great enough. It is know from the assumption that continues iteration shall result in not researching better points, so we can select other points as the initial points to calculate.

**Correction example of friction resistance coefficient**

Some ideal water injection pipeline network of the example is composed of two water injection stations, 16 nodes, 24 pipelines and 9 loops, the nodes with 2, 15 numbers are the positions of the pump station. Simulating three working conditions of the pipeline network: the pump station is totally opened and the pump station is closed successively. The simplified drawing of the water injection pipeline network is as shown in figure 1, and the base data is as shown in table 1.

![Simplified drawing of pipeline network](image)

**Table 1: correction result of friction resistance coefficient**

<table>
<thead>
<tr>
<th>Pipeline number</th>
<th>Actual friction resistance coefficient</th>
<th>Calculated friction resistance coefficient</th>
<th>Calculated friction resistance coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>85</td>
<td>85</td>
<td>13</td>
</tr>
<tr>
<td>2</td>
<td>85</td>
<td>83.3</td>
<td>14</td>
</tr>
<tr>
<td>3</td>
<td>95</td>
<td>93.5</td>
<td>15</td>
</tr>
<tr>
<td>4</td>
<td>85</td>
<td>85</td>
<td>16</td>
</tr>
<tr>
<td>5</td>
<td>85</td>
<td>85.8</td>
<td>17</td>
</tr>
<tr>
<td>6</td>
<td>95</td>
<td>91.9</td>
<td>18</td>
</tr>
<tr>
<td>7</td>
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<td>19</td>
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<tr>
<td>8</td>
<td>85</td>
<td>85</td>
<td>20</td>
</tr>
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<td>84.9</td>
<td>23</td>
</tr>
<tr>
<td>12</td>
<td>105</td>
<td>106</td>
<td>24</td>
</tr>
</tbody>
</table>
The correction result is satisfying by comparing the correction result of friction resistance coefficient, and the average value of the correction value of the friction resistance coefficient and the value of the actual friction resistance coefficient is 1.24. The reason of the difference between the actual friction resistance coefficient and the calculated friction resistance coefficient is the multiple solutions of the optimization problem. The target function value corresponding to the calculated friction resistance coefficient reaches $10^{-3}$, that is to say the calculated pressure of each node of each working condition is the same as the actual pressure. The optimization algorithm is improved further to improve the optimization performance, and it requires deeper research.

CONCLUSION

Based on the filled function method, the friction resistance coefficient of the pipeline network pipeline is corrected. Under the precondition that the pressure of each node is known, the working condition number is confirmed according to the number of the water injection pipeline network pump station. The constraint condition is the value scope of the friction resistance coefficient. The node pressure under many working conditions is taken as the data to set up the correction optimization mathematical model of pipeline friction resistance coefficient under many working conditions. A one-parameter filled function is given based on the former filled function. The correction optimization mathematical model of the friction resistance coefficient is solved by the given auxiliary function to obtain better correction result.

REFERENCES