

Research Article

Simulating Calculation of Oilfield Water Injection System

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Abstract: The text introduces the base theory of pipeline network hydraulic calculation, analyzes how to set up the mathematical model solving the node pressure by the continuous equation and pressure drop equation and discusses the solving method of the mathematical model.

Keywords: Water injection pipeline, Friction coefficient, Global optimization

INTRODUCTION

The water injection system is a hydraulic system composed of links, such as water injection station, water injection well, water allocating station and the like as well as the pipelines connecting each link[1,3-5]. In the water injection system, the water is supplied by each water source, the water in each water supply source is processed to meet the production requirement on the water injection quality, and then the water is conveyed to the water tank in the water injection station, after being pressurized, the water is injected to the water injection pipeline network and reaches to the water allocating station, and then the water flows to the water injection well from the water allocating station, finally, the water is injected to the stratum.

OILFIELD WATER INJECTION SYSTEM

The oilfield water injection system [2] is a huge system. Generally, in order to reduce the dimension of the system equation and retain the main information of the former system, the simplified policy is adopted as follows:

- The injection allocation flow of the water injection node directly connected with the water injection main line is simplified to the water injection main line node connected with the water injection node, namely, the water injection main line is simplified by connecting to the water injection node directly.
- The injection allocation flow of the water injection branch is simplified to the water injection branch node connected with the water injection branch, namely, the water injection branch node is simplified.

BASE EQUATION OF PIPE NETWORK HYDRAULIC CALCULATION[4]

Assuming that P is the number of the middle section of the pipeline network, S is the number of water allocation source, J is node number, including the water allocation source node, L is loop number, the unknown quantity is $P q_{ij}$ and h_{ij} , S water allocation source water supply and J water pressure H_i , sum to $P+ J+ S$ unknown quantities.

Node equation

What is node equation? As far as any node, the flow flowing to the node must be equal to that flowing out from the node to meet the flow balance of the node, and the mathematical expression is as follows:

$$\sum q_{ij} + Q_i = 0 \quad (1)$$

In which, Q_i is the flow of node i , and q_{ij} is the pipe flow from node i to node j .

Energy equation

The energy equation shows the relation that the head loss sum of each pipeline in each link of the pipe network is equal to 0, and it is the energy balance equation of the closed loop and shown as follows:

$$\sum_1^L h_{ij} - \Delta H_k = 0 \tag{2}$$

In which, h_{ij} is the head loss of the pipe section from node i to node j , and ΔH_k is the closed difference of closed different of base loop k or water pressure difference generated by increasing and decreasing pressure.

Pressure drop equation

The relation between the head loss of the pipeline and the water pressure of the two ends at the two ends is called as the pressure drop equation. When calculating the pipeline network, the local resistance loss is not counted. If counting the head loss along the process, the relation between the flow and the head loss is as shown as follows:

$$h_{ij} = H_i - H_j = s_{ij} q_{ij}^n \tag{3}$$

In which, H_i, H_j are water pressure elevation of the nodes i, j at the two ends of the pipeline, s_{ij} is the friction resistance of the pipeline, and $n = 1.852 \sim 2$.

Virtual loop equation

The relation shown by the head loss with relation line between the water allocation sources is called as the virtual loop equation which is as shown as follows:

$$F(Q)_1 - F(Q)_k = (\sum h)_{1-k} \tag{4}$$

In which, $F(Q)$ shows the flow and hydraulic characteristics of water allocation sources.

In above equation, the pressure drop equation number is $P - L$, the node equation is J , the energy equation is L , the virtual loop equation is $S - 1$, and the total number of the equation is $J + P - L + L + S - 1 = P + J + S - 1$. All pipeline flow and the head loss can be obtained by the equations, and the H_i value cannot be confirmed.

There are three solving methods: unlink equation method, un-node equation method and pipeline solving pressure drop equation method. The leading thought of the unlink equation method is to amend the pipeline flow gradually to meet the energy equation finally. The greatest advantage of the un-node equation is little workload of inputting data, multiple work is finished by the computer, so it is wide calculation method at preset. The pipeline solving pressure drop equation method solves the water pressure of unknown nodes according to the known node pressure. The main distinction of the three methods is different unknown quantities and the pipeline network equation composed of three equations.

SIMULATING CALCULATION OF OILFIELD WATER INJECTION SYSTEM

The simulating calculation of oilfield water injection system is to calculate the pressure of each node in the system and the flow of the pipeline under the precondition that the length, diameter, friction resistance coefficient and node flow of each pipeline in the pipeline system are known [2]. The following simple ideal pipeline network is taken as the example.

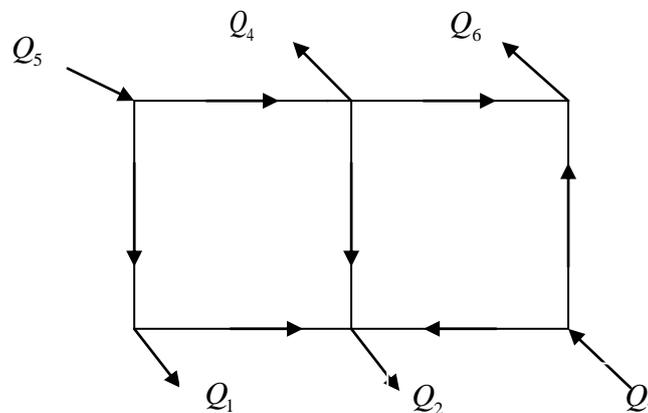


Fig. 1: Ideal pipeline network drawing

Simulating total equation of system

For the continuous equation, each node i is set up:

$$\sum q_{ij} + Q_i = 0 \quad i=1, 2, 3, 4, 5, 6. \tag{5}$$

Q_i is the flow of i node and conforms to the regulation with negative inflow and positive flowing out.

q_{ij} meets

$$H_i - H_j = s_{ij} |q_{ij}|^{n-1} q_{ij} \tag{6}$$

Or

$$q_{ij} = \text{sgn}(H_i - H_j) \left(\frac{|H_i - H_j|}{s_{ij}} \right)^{\frac{1}{n}} \tag{7}$$

In $s_{ij} = \frac{10.667l_{ij}}{C_{ij}^{1.852} d_{ij}^{4.87}}$, $n=1.852$, l_{ij} is the length of the pipeline, unit is meter, d_{ij} is the diameter of the pipeline,

unit is meter, and C_{ij} is the friction resistance coefficient of the pipeline.

$H_i - H_j = s_{ij} |q_{ij}|^{n-1} q_{ij}$ can obtain

$$q_{ij} = \frac{1}{s_{ij} |q_{ij}|^{n-1}} (H_i - H_j)$$

Recording as

$$B_{ij} = \frac{1}{s_{ij} |q_{ij}|^{n-1}} \tag{8}$$

So

$$q_{ij} = B_{ij} (H_i - H_j)$$

Substituting the above formula into the equation

$$\sum B_{ij} (H_i - H_j) + Q_i = 0 \quad i=1, 2, 3, 4, 5, 6. \tag{9}$$

Concrete form:

$$\begin{cases} B_{12}(H_1 - H_2) + B_{15}(H_1 - H_5) & + Q_1 = 0 \\ B_{21}(H_2 - H_1) + B_{24}(H_2 - H_4) + B_{23}(H_2 - H_3) & + Q_2 = 0 \\ B_{32}(H_3 - H_2) + B_{36}(H_3 - H_6) & + Q_3 = 0 \\ B_{42}(H_4 - H_2) + B_{45}(H_4 - H_5) + B_{46}(H_4 - H_6) & + Q_4 = 0 \\ B_{51}(H_5 - H_1) + B_{54}(H_5 - H_4) & + Q_5 = 0 \\ B_{63}(H_6 - H_3) + B_{64}(H_6 - H_4) & + Q_6 = 0 \end{cases} \tag{10}$$

Cleared up and obtained

$$\left\{ \begin{array}{l} (B_{12} + B_{15})H_1 - B_{12}H_2 - B_{15}H_5 = -Q_1 \\ -B_{21}H_1 + (B_{21} + B_{23} + B_{24})H_2 - B_{23}H_3 - B_{24}H_4 = -Q_2 \\ -B_{32}H_2 + (B_{32} + B_{36})H_3 - B_{36}H_6 = -Q_3 \\ -B_{42}H_2 + (B_{42} + B_{45} + B_{46})H_4 - B_{45}H_5 - B_{46}H_6 = -Q_4 \\ -B_{51}H_1 - B_{54}H_4 + (B_{51} + B_{54})H_5 = -Q_5 \\ -B_{63}H_3 - B_{64}H_4 + (B_{63} + B_{64})H_6 = -Q_6 \end{array} \right. \quad (11)$$

$$\text{Recording } \tilde{\mathbf{B}} = \begin{bmatrix} B_{12} + B_{15} & -B_{12} & 0 & 0 & -B_{15} & 0 \\ -B_{21} & B_{21} + B_{23} + B_{24} & -B_{23} & -B_{24} & 0 & 0 \\ 0 & -B_{32} & B_{32} + B_{36} & 0 & 0 & -B_{36} \\ 0 & -B_{42} & 0 & B_{42} + B_{45} + B_{46} & -B_{45} & -B_{46} \\ -B_{51} & 0 & 0 & -B_{54} & B_{51} + B_{54} & 0 \\ 0 & 0 & -B_{63} & -B_{64} & 0 & B_{63} + B_{64} \end{bmatrix}$$

$$\tilde{\mathbf{H}} = (H_1, H_2, H_3, H_4, H_5, H_6)^T$$

$$\tilde{\mathbf{Q}} = (-Q_1, -Q_2, -Q_3, -Q_4, -Q_5, -Q_6)^T$$

The equation set is written as:

$$\tilde{\mathbf{B}}\tilde{\mathbf{H}} = \tilde{\mathbf{Q}} \quad (12)$$

The flow sum of each node in the water injection pipeline network system is 0, namely,

$$Q_1 + Q_2 + Q_3 + Q_4 + Q_5 + Q_6 = 0 \quad (13)$$

If the equation set (11) has infinitely many solutions, the pressure of one reference point must be given when being calculated in order to enable the equation set to have the unique solution. H_6 is used as the pressure of the known reference point, the last equation is removed, and the following equation set is obtained finally:

$$\left\{ \begin{array}{l} (B_{12} + B_{15})H_1 - B_{12}H_2 + B_{15}H_5 = -Q_1 \\ -B_{21}H_1 + (B_{21} + B_{23} + B_{24})H_2 - B_{23}H_3 - B_{24}H_4 = -Q_2 \\ -B_{32}H_2 + (B_{32} + B_{36})H_3 = -Q_3 + B_{36}H_6 \\ -B_{42}H_2 + (B_{42} + B_{45} + B_{46})H_4 - B_{45}H_5 = -Q_4 + B_{46}H_6 \\ -B_{51}H_1 - B_{54}H_4 + (B_{51} + B_{54})H_5 = -Q_5 \end{array} \right. \quad (14)$$

$$\mathbf{B} = \begin{bmatrix} B_{12} + B_{15} & -B_{12} & 0 & 0 & -B_{15} \\ -B_{21} & B_{21} + B_{23} + B_{24} & -B_{23} & -B_{24} & 0 \\ 0 & -B_{32} & B_{32} + B_{36} & 0 & 0 \\ 0 & -B_{42} & 0 & B_{42} + B_{45} + B_{46} & -B_{45} \\ -B_{51} & 0 & 0 & -B_{54} & B_{51} + B_{54} \end{bmatrix}$$

$$\mathbf{H} = (H_1, H_2, H_3, H_4, H_5)^T$$

$$\mathbf{Q} = (-Q_1, -Q_2, -Q_3 + B_{36}H_6, -Q_4 + B_{46}H_6, -Q_5)^T$$

Under the precondition that the pressure H_6 of the reference point is known, the new nonlinear equation set is written as:

$$BH = Q \quad (15)$$

Calculation method for simulating total equation of system

The equation set (15) has the unique solution and is converged when being solved by the linear simple iteration method which has the following process:

- (1) Giving the pressure value of the reference point;
- (2) Pre-estimating node pressure $H^{(0)}$ of ne group as the initial value, giving ε precisely, and $k = 0$;
- (3) Confirming q_{ij} by node pressure $H^{(k)}$ and formula (7), and then confirming the coefficient matrix B ;
- (4) Obtaining the pressure value $H^{(k+1)}$ of each node by the simultaneous equations method (15);
- (5) If the precision meets

$$\|H^{(k+1)} - H^{(k)}\| \leq \varepsilon, \quad i = 1, \dots, N \quad (16)$$

If so, transferring to (6), if not, $k = k + 1$, transferring to (3);

- (6) Ordering $H^* = H^{(k+1)}$, obtaining the pressure value of each node, ending the iteration.

CONCLUSION

The text introduces the base theory of pipeline network hydraulic calculation, analyzes how to set up the mathematical model solving the node pressure by the continuous equation and pressure drop equation and discusses the solving method of the mathematical model. During the actual application, some fuzzy and random parameters are still processed carefully, and it needs further research and discussion.

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