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Research Article

Hybrid Modeling of the Urban Traffic Flow

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Abstract: Nowadays, the development of tools for modeling and simulating road traffic flow becomes more and more a necessity. This is reflected by the development of many road traffic flow models that can reproduce some traffic phenomena. The objective of this paper is to present a hybrid model based on two models developed independently. The hybrid scheme is based on the coupling scheme developed by Bourrel. Coupled models are the macroscopic LWR model (based on its resolution by GODUNOV's method) and a microscopic car following model presented in this paper. The proposed model is validated by simulation.
Keywords: Traffic modeling, Macroscopic model, Microscopic model, Hybrid model.

INTRODUCTION

For a long time, urban transportation in sub-Saharan Africa, has been placed among the non-priority sectors and become in a recent years a major concern for authorities to control and develop the sector. Thus, the provision of tools adapted to the context and allowing the modeling traffic flow phenomena is a necessity. This modeling, which consists to a description of the traffic evolution over time and space, can help to understand these phenomena.

There are several approaches to model the traffic flow. These approaches can be classified into two major families which are: the macroscopic approach and the microscopic approach [1].

The microscopic approach focuses on the interactions between the vehicles considered individually. The models resulting from this approach, explicitly represent the dynamic states of vehicles [2]. In contrast, the macroscopic approach describes the traffic flow in a comprehensive manner [3].

Each of these types of approaches is adapted to specific situations. For example, macroscopic approaches fail in describing transitional phases in traffic. The microscopic models allow a clear view of these transitional phases, but become quickly complicated when a large network is considered. In this case the computational time is very long compared to macroscopic models.

The aim of this paper is to propose a model of urban transport system, which can represent singular phenomena which may be the origin of disturbances in the network. It is well known that the macroscopic LWR model is known for its ability to describe the overall traffic dynamics [4]. But this model does not highlight several types of singularities which have an impact on the traffic.

Thus the development of a hybrid model will first allow the overall traffic dynamics to be represented by the LWR model by solving it with the Godunov's method, and singular elements to be represented by using a microscopic traffic flow model that will be presented.

In the rest of this paper, the LWR model and its resolution by the Godunov's method will be presented, after, the microscopic model will be presented. The coupling scheme which is based on the scheme proposed by Bourrel [5] with some modifications will be described. To conclude, a presentation of some simulation results will be made.

THE LWR MODEL

In this model the traffic is represented as a continuous fluid characterized by average quantities depending on time and space. These quantities are the flow $q(x,t)$, the concentration or density $k(x,t)$ and the speed flow $v(x,t)$.

The fundamental equation of the model is the conservation equation (equation 1), which was first used for modeling traffic flow in 1955, by Lighthill and Whitham, and independently by Richards in 1956 [6, 7]. This equation expresses the fact that the number of vehicles in a road section at a time $t+dt$ is equal to the number of vehicles in this section at time t, to which we add the number of vehicles entered during dt, minus the number of vehicles exited during dt.

$$\frac{\partial k(x,t)}{\partial t}+\frac{\partial q(x,t)}{\partial x}=0 \qquad (1)$$

The flow speed is defined as the ratio of flow on the density.

$$v(x,t)=\frac{q(x,t)}{k(x,t)} \qquad (2)$$

The LWR model is supplemented by an equilibrium fundamental relationship which varies according to parameters of studied network.

Using the macroscopic definition of flow speed, it is possible to derive an equilibrium relationship between flow and density. This relationship is deduced from experimental observations, and is represented by a diagram called fundamental diagram [8] (Fig. 1).

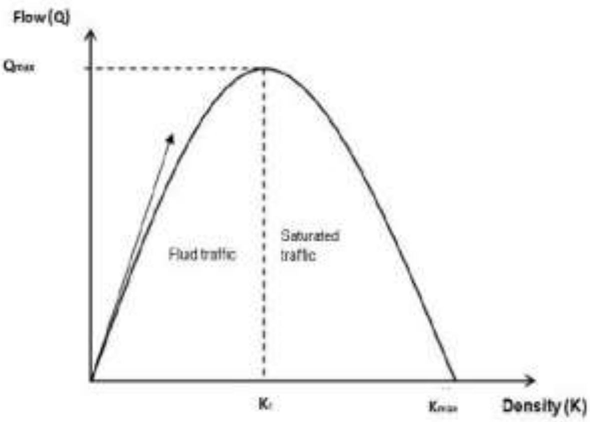

Fig. 1: Example of parabolic fundamental diagram

The main parameters of a fundamental diagram are:

- maximum density, denoted $K_{\max}$,
- the free speed, denoted V_l given by the slope at the origin of the fundamental diagram,
- the critical concentration K_c between the fluid and saturated traffic conditions,
- the maximum flow $Q_{\max}$ or capacity of the section studied,
- and critical speed V_c or speed of vehicles at critical concentration.

In the LWR model, we assumed that the system is always in equilibrium. Hence, the speed is function of the concentration.

$$v(x,t)=V_{eq}(k(x,t)) \qquad (3)$$

RESOLUTION OF THE LWR MODEL BY GODUNOV'S APPROACH

In the Godunov's scheme, each road section is divided into cells of length Δx (Fig. 4). We denote by δx_i the length of a cell whose interfaces are represented by points x_i and x_{i+1}, respectively input and output of the cell noted by:

$$C_i=[x_i,x_{i+1}] \qquad (4)$$

The equation (1) applied to the cell Ci leads to the following one:

$$\int_{x_i}^{x_{i+1}}\left(\frac{\partial k(x,t)}{\partial t}+\frac{\partial q(x,t)}{\partial x}\right)dx=0 \qquad (5)$$

After integration, we have:

$$\frac{d}{dt}\int_{x_i}^{x_{i+1}}k(x,t)dx+q(x_{i+1},t)-q(x_i,t)=0 \qquad (6)$$

The average density $k_i(t)$ of cell C_i is introduced:

$$k_i(t)=\frac{1}{\Delta x_i}\int_{x_i}^{x_{i+1}}k(x,t)dx \qquad (7)$$

We denote by $Q_i(t)=q(x_i,t)$ the flow on the interface x_i at time t. Considering the Euler approximation of the first order time derivative, the conservation equation in the cell C_i (equation 5) leads to the following equation:

$$\Delta x_i\,\frac{k_i(t+\Delta t)-k_i(t)}{\Delta t}+Q_{i+1}(t)-Q_i(t)=0 \qquad (8)$$

The parameters of the numerical scheme (equation 8) are defined by supply and demand functions (Fig. 3).

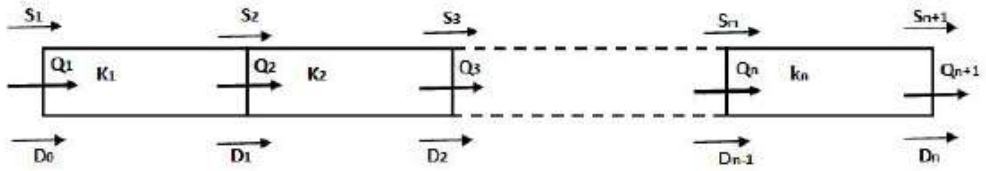

Fig. 2: Subdivision of a section of road to cells

The supply function denoted by $S_i(k)$ is the maximum number of cars that can enter the cell i during the time interval Δt.

In fluid traffic situation it can be observed that this function is equal to the maximum rate that can enter the cell, while in congested traffic situation, this function is given by an equilibrium relationship $Q_i(k)$ deduced from the fundamental diagram.

$$S_i(k) = \begin{cases} Q_{\max} & if \ k \le K_c \\ Q_i(k) & if \ k > K_c \end{cases} \quad (9)$$

The demand function denoted $D_i(k)$, is the maximum number of cars wishing to go out during the same interval of time.

In situations of fluid traffic, this function is given by an equilibrium relation $Q_i(k)$, derived from the fundamental diagram, while in congested traffic situation, this function is equal to the maximum rate that can leave the cell.

$$D_i(k) = \begin{cases} Q_i(k) & if \ k \le K_c \\ Q_{\max} & if \ k > K_c \end{cases} \quad (10)$$

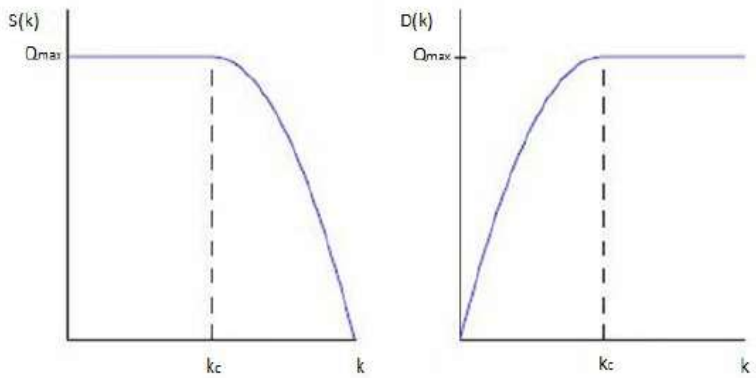

Fig. 3: Supply diagram at left and demand diagram at right

From the determination of the maximum number of vehicles that can enter a cell C_{i+1} and the maximum number of vehicles that can leave the upstream cell C_i for the interval time Δt, we can calculate the average flow $Q_i(t+\Delta t)$ to the point x_i separating the two cells. This flow is the minimum between the supply of downstream cell C_{i+1} and the demand of the upstream cell C_i at this point during this interval time. Given the equation 8, we have:

$$\begin{cases} Q_i(t \to t+\Delta t) = \min(D(k_i), S(k_{i+1})) \\ k_i(t+\Delta t) = k_i(t) + \dfrac{\Delta t}{\Delta x}(Q_{i-1}(t \to t+\Delta t) - Q_i(t \to t+\Delta t)) \end{cases} \quad (11)$$

Hence, from the initial conditions of each cell, we can determine the evolution of traffic flow by successive time steps.

For the stability of this scheme, the time step and the length of a cell should be chosen so that the computed solution to an interface does not interfere with the rest. So each cell obeys to the following CFL condition (Courant Friedrichs-Lewy)

$$\frac{\Delta x}{\Delta t} \geq V_l \qquad (12)$$

with Δx the length of a cell, Δt the observation time.

The resolution algorithm is given below:

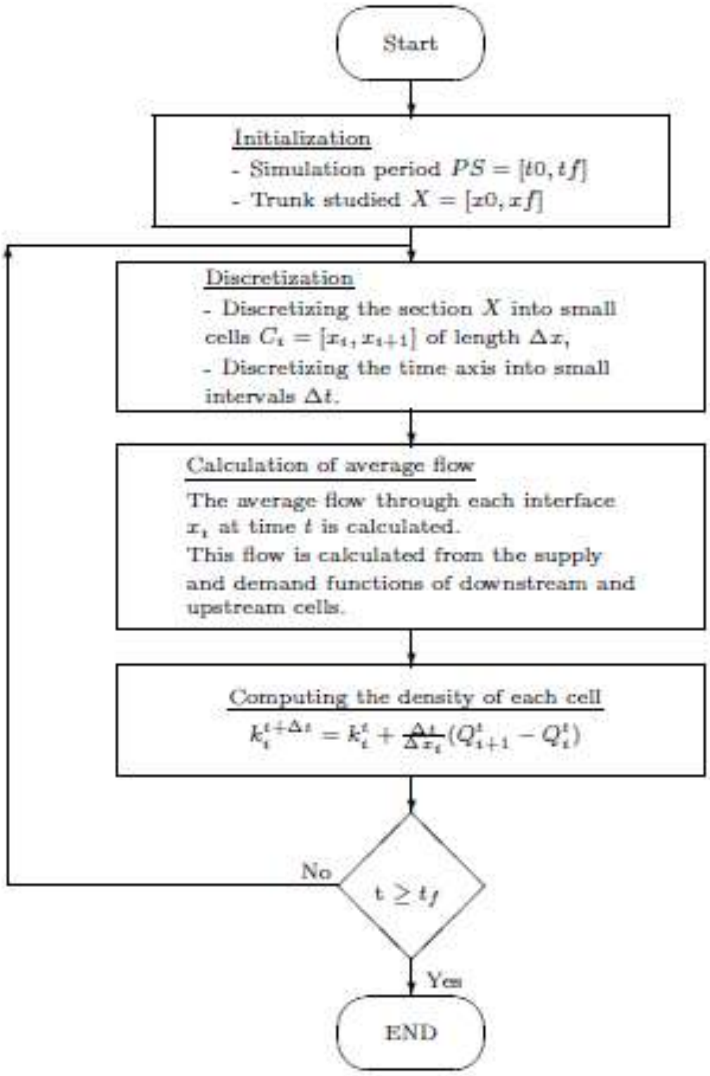

Fig. 4: Algorithm for solving the LWR model by Godunov's method

The Godunov's scheme allows a simple modeling of the traffic flow. It is also a model well suited to urban environments. However the Godunov's method does not reproduce the transition phases related to incidents or vehicle's starting at the end of a red light, either in acceleration or deceleration.

For example, in the case of a spatial discontinuity (x0 in Fig. 5) it can be noticed that, for the same speed, traffic conditions are transferred from the equilibrium state corresponding to this flow on the upstream fundamental diagram to that corresponding to the same flow on the downstream diagram. The speed is discontinuous at this point.

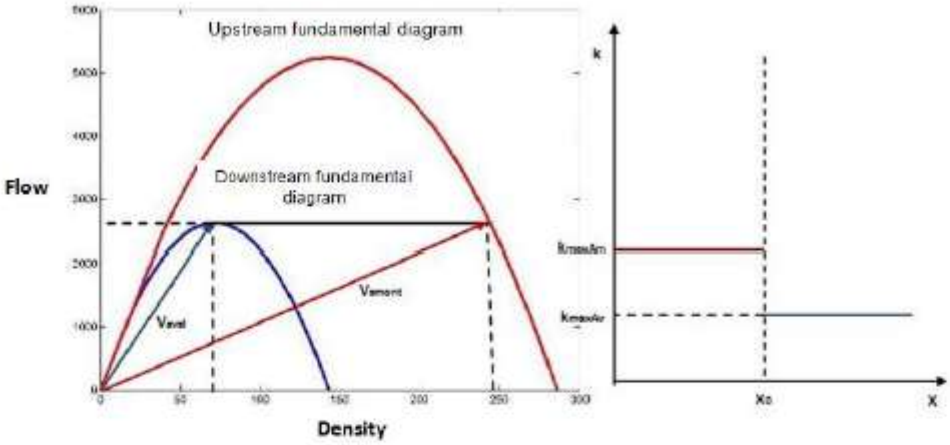

Fig. 5: Case of spatial discontinuity: velocity change at the point of discontinuity

THE MICROSCOPIC CAR FOLLOWING MODEL

In this model, a vehicle i at a position x_i at time t is characterized by its speed $v_i(\mathrm{x},\ \mathrm{t})$ and its acceleration $a_i(\mathrm{x},\ \mathrm{t})$.

Lets considering a vehicle $i+1$ (follower) at position x_{i+1} behind a vehicle i at time t. To represent the dynamical behavior of the driver/vehicle pair $i+1$ at time $t+dt$, the acceleration of this vehicle is defined as a function of the relative speed and spacing between vehicles.

$$a_{i+1}(t) = f(v_i(t) - v_{i+1}(t), x_i(t) - x_{i+1}(t)) \quad (13)$$

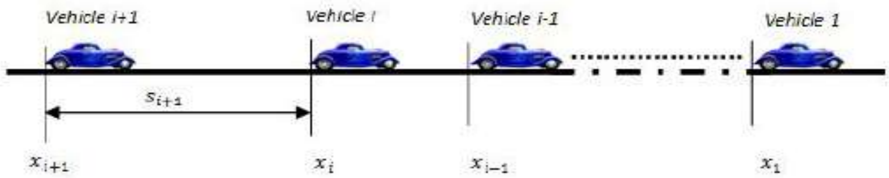

Fig. 6: Vehicles in a section road

Let's take $dt = T_1$. The action of the driver at the control levers of his vehicle at time $t+T_1$ is proportional to the spacing between the position of the vehicle $i+1$ (x_{i+1}) and the position x_i of the vehicle i at time t (Fig. 7).

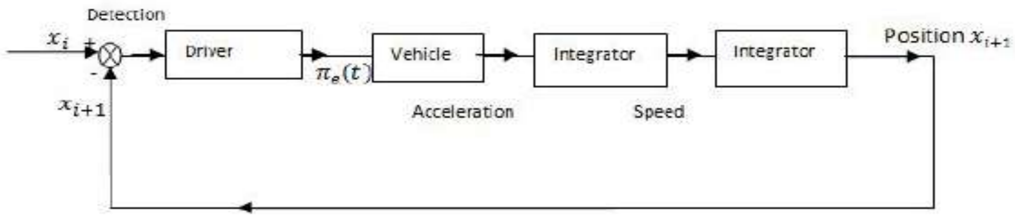

Fig. 7: Graphical basic scheme of driver-vehicle pair

The model of the driver's behavior includes the following functions:

- Detection of the difference between the actual and desired spacing,
- Decision-making and
- Action on the control levers..

Hence, a graphical model of the driver can be represented at Fig. 8, in which the action of the driver is linked to the spacing between the vehicle $i+1$ at position x_{i+1} and the vehicle i at position x_i at time t, by the constant parameter g (equation 14).

$$\pi_{c(i+1)}(t) = g(x_i(t) - x_{i+1}(t)) \qquad (14)$$

where g is a reaction parameter and $\pi_{c(i+1)}$ the thrust or braking related to driver's behavior.

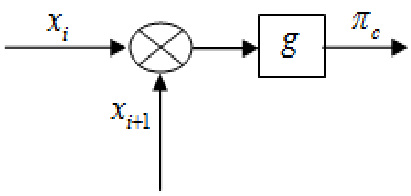

Fig. 8: Base model of driver

In order to take into account the phenomena of acceleration and deceleration, it is necessary to anticipate a slowdown or acceleration zone. This anticipation will help to prevent the too fast advancing of vehicles. Thus, in addition to the delay time T_1 corresponding to the reaction time of drivers, the integration of a diverter function for the problem of anticipation, offers a new dynamic relationship between $\pi_{c(i+1)}(t)$ and the effective action $\pi_{e(i+1)}(t)$, represented by equation 15.

$$\pi_{e(i+1)}(t) = \pi_{c(i+1)}(t - T_1) + T_d \frac{d\pi_{c(i+1)}(t - T_1)}{dt} \qquad (15)$$

where T_d is the response time of the diverter.

The acceleration are related to the effective action by the following relation:

$$a_{i+1}(t) = \alpha \frac{d}{dt} \pi_{e(i+1)}(t) - \tau \frac{d}{dt} a_{i+1}(t) \qquad (16)$$

Where α is a constant parameter and τ the response time of the motor.

From Equations 15 and 16, we deduce the acceleration, represented by equation 17.

$$a_{i+1}(t) = -\frac{1}{\tau} v_{i+1}(t) + \frac{\alpha T_d g}{\tau}(v_i(t - T_1) - v_{i+1}(t - T_1)) + \frac{\alpha g}{\tau}(x_i(t - T_1) - x_{i+1}(t - T_1)) \qquad (17)$$

Finally the car following model is represented by the following system 18.

$$\begin{cases} a_{i+1}(t) = -\frac{1}{\tau} v_{i+1}(t) + \frac{\alpha T_d g}{\tau}(v_i(t - T_1) - v_{i+1}(t - T_1)) + \frac{\alpha g}{\tau}(x_i(t - T_1) - x_{i+1}(t - T_1)) \\ \frac{d}{dt} v_{i+1}(t) = a_{i+1}(t) \\ \frac{d}{dt} x_{i+1}(t) = v_{i+1}(t) \end{cases} \qquad (18)$$

In this model, the acceleration depends not only on the spacing between the following car and its leader and the difference between the two speeds, but also on the information on the speed of the following car at time t.

The equilibrium is characterized by the situation at which vehicles travel at the same speed. At this equilibrium we must have:

$$v_{i+1}(t) = v_i(t) = V_{eq}$$
$$\frac{d}{dt} v_{i+1}(t) = 0$$
$$x_i(t) - x_{i+1}(t) = S_{eq}$$

Then we have:

$$\begin{cases} -\frac{1}{\tau}v_{i+1}(t)+\frac{\alpha T_d k}{\tau}(v_i(t-T_1)-v_{i+1}(t-T_1))+\frac{\alpha k}{\tau}(x_i(t-T_1)-x_{i+1}(t-T_1))=0 \\ \frac{d}{dt}v_{i+1}(t)=a_{i+1}(t)=0 \\ \frac{d}{dt}x_{i+1}(t)=v_{i+1}(t)=V_{eq} \end{cases} \quad (19)$$

From equation 19, we get the following relation:

$$S_{eq}\ \frac{V_{eq}}{\alpha.g} \quad (20)$$

To determinate the parameter g from equation 20, we use the definition of the fundamental diagram. This diagram provides the fundamental equilibrium relationship between concentration and speed.

The concentration k_{eq} is related to S_{eq} by the equation 21:

$$S_{eq}=\frac{1}{k_{eq}} \quad (21)$$

The calculated maximum speed should not be greater than that allowed at the fixed free maximum concentration $K_{l\max}$.

For example, with the equilibrium relationship given above, we can define:

- $S_{\min}=\frac{1}{K_{\max}}$: The minimum equilibrium distance between two vehicles,
- $S_{\max}=\frac{1}{K_{l\max}}$: The minimum equilibrium distance for which vehicles run at full speed.

Then we must have:

$$g.S_{\min}\le \pi_c(\mathrm{t})\le g.S_{\max} \quad (22)$$

The field observations show that acceleration and deceleration are not symmetric problems. The acceleration response time is greater than the response time of deceleration. Thus we define two response times τ_1 and τ_2 such as $\tau_1<\tau_2$. And the term of the acceleration or deceleration is given by the equation 23.

$$\begin{cases} a_{i+1}(t)=-\frac{1}{\tau_1}v_{i+1}(t)+\frac{\alpha T_d k}{\tau_1}(v_i(t-T_1)-v_{i+1}(t-T_1))+\frac{\alpha k}{\tau_1}(x_i(t-T_1)-x_{i+1}(t-T_1)) \quad \text{Deceleration} \\ a_{i+1}(t)=-\frac{1}{\tau_2}v_{i+1}(t)+\frac{\alpha T_d k}{\tau_2}(v_i(t-T_1)-v_{i+1}(t-T_1))+\frac{\alpha k}{\tau_2}(x_i(t-T_1)-x_{i+1}(t-T_1)) \quad \text{Acceleration} \end{cases} \quad (23)$$

COUPLING THE TWO MODELS

In order to take into account transitional phases not taken by the LWR model, a hybrid model is presented here. This model is used to represent some part of network by a microscopic model while the other parts of the network are represented by the LWR model. The coupling scheme we use must allow correct transmission of information from one model to another

Let's consider a road section with a given length. Applying a hybrid model on this road section consist of dividing the road section into areas where, either the microscopic model or the LWR model are applied.

In the example in Fig. 9, the road is divided into three zones, and the LWR model with its resolution by the

Godunov's scheme is applied in two zones separated by a zone in which the car following model is applied.

At interfaces we define transitional cells. These cells are virtual and allow information exchanges between the two models at the interfaces. The interest of these transition cells is to allow satisfactory boundary conditions to each model, and also enable a gradual transfer of information from one model to another.

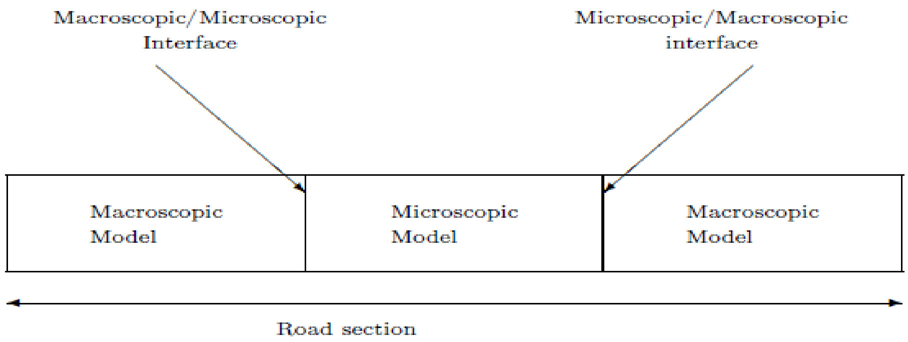

Fig. 9: Coupling scheme

Let a certain instant t where the state of the system is known, and Δt_{God} and Δt_{mic} the temporal discretisation steps applied respectively to the macroscopic model and the microscopic model. The coupled models are discrete time and the problem is to determine the state of the system at $t+\Delta t_{God}$.

By imposing $\Delta t_{God} = N\Delta t_{mic}$, we will have instants where the two models know the state of the system simultaneously, with N a positive integer. Thus information is exchanged at the interfaces at each time step of the macroscopic model.

The principle of this scheme is divided into four stages:

- Step 1: We calculate for the Godunov's model, the upstream demand of the microscopic area during a time step, and the downstream supply
- Step 2: After we translate these constraints into vehicular constraints for cell transition through the generation of vehicles at the interface Macroscopic/Microscopic and determination of the trajectory of the first vehicle at the interface Microscopic/Macroscopic.
- Step 3: we evolve the whole vehicle of the microscopic, based on the car following model, from t to $t+\Delta t_{God}$ per successive time step Δt_{mic}, taking into account the constraints at interfaces.
- Step 4: we deduce from these trajectories, the density of these cells from t to $t+\Delta t_{God}$.

These flows are used as boundary conditions for the macroscopic model and allow the calculation of its state at $t+\Delta t_{God}$.

At the interfaces of the microscopic area, we impose the demand and supply of macroscopic model as boundary conditions.

Thus, the microscopic model should provide to the macroscopic model, downstream supply at interface Macroscopic/Micoscopic, and upstream demand of the interface Microscopic/Macroscopic.
Conversely, the macroscopic model must provide to the microscopic model, the generation times of vehicles at the interface Macroscopic/Microscopic, and the trajectory of the first vehicle at upstream interface Microscopic/Macroscopic.

Flows are calculated retrospectively: the growth of vehicles from t to $t+\Delta t_{God}$ gives the supply and demand of transition cells.

The generation of vehicles at the interface Macroscopic/Microscopic must satisfy both the demand on the macroscopic model and downstream traffic conditions. It is assumed that the generation of vehicles at the entrance of each section will be made uniformly.

We define:

- The minimum time interval between two generations of vehicle at the interface Macroscopic/Microscopic CI, as the inverse of the demand imposed by the macroscopic model.

$$CI = \frac{1}{D(k)}$$

- Nsas: entry transition sas. This sas is designed to materialize at the end of time step Δt_{God}, the presence of a portion of vehicle not sufficient to generate a vehicle and to reduce the oscillations in the hybrid model.

To determine the generation times of vehicles at the interface Macroscopic/Microscopic, we calculate the first moment of generation, to complete the portion of vehicle already present in the cell, given by $(1-\text{Nsas})\text{CI}$.

The predicted generation times of other vehicles are calculated using time spacing equal to CI.

The creation of a vehicle will be made only, if the spacing between the last vehicle and the entrance of the microscopic area is more than or equal to the equilibrium spacing corresponding to the speed of the last vehicle, if this speed is less than or equal to the speed calculated from the demand at the entrance of the microscopic area.

The trajectory of a vehicle coming out from the microscopic area must meet downstream traffic conditions. We also determine the exit times of vehicles. We define NsasS as the output transition sas.

SIMULATIONS WITH THE HYBRID MODEL PROPOSED

In this section we will discuss different scenarios to study the behavior of the model, especially with regard to the spread of information depending on different parameters. In simulation, a 6km stretch of road is used. A time step $\Delta t_{\text{God}} = 5s$ is chosen. The section of road is divided into cells of length $\Delta x = 88{,}2m$. The cells are numbered from the upstream to downstream; the cells 58 and 59, located between 4km and 4.1400km are represented by the microscopic model.

Scenario 1: Propagation of congestion to upstream

For this purpose, we take an equilibrium condition where demand at the network entry is set to 0.4167veh/s and the supply at the network output is set to 0.2778veh/s. The initial concentration of cells is set to 50veh/km.

Through the results of presented in figure 10, there is a congestion propagating upstream. In this figure we have represented the evolution of the traffic flow on this stretch of road in the plane (x, t).

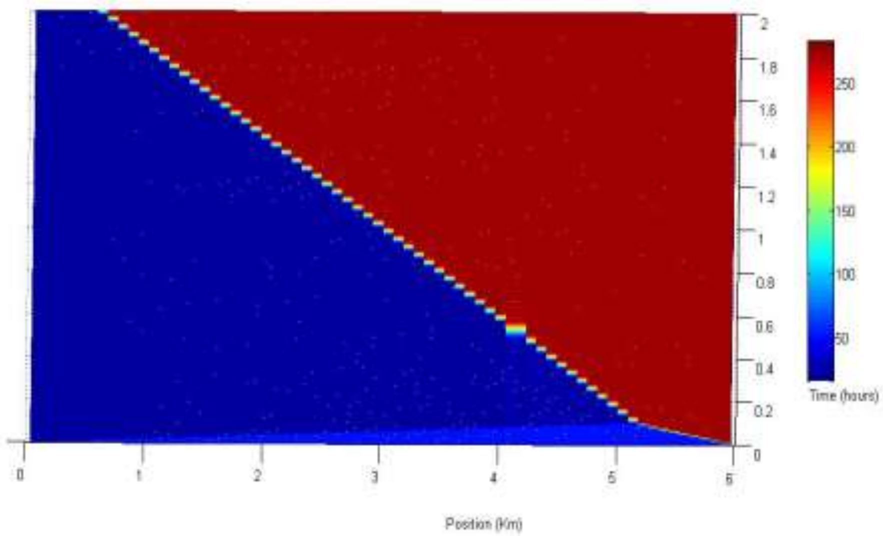

Fig. 10: Propagation of congestion to upstream: plane (x, t)

Scenario 2: Reduction of congestion

This time we are in an equilibrium situation where demand at the input of the network is fixed to 0.3333veh/s and supply at the output of the network is set to 0.5000veh/s. The initial concentration of cells is set at 260veh/km.

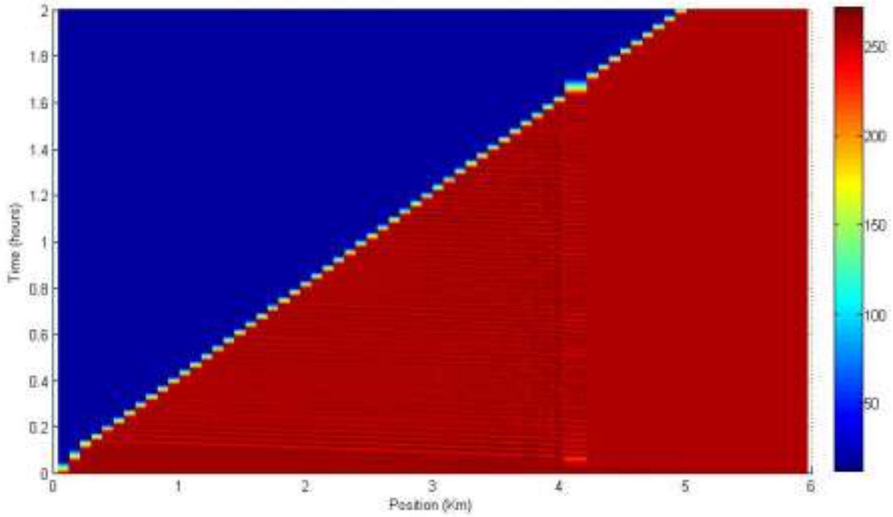

Fig. 11: Reduction of congestion in the hybrid model

Through the results presented in figure 11, the formation of a range which propagates through the microscopic model can be observed without distortion. In this figure we have represented the evolution of the traffic flow on the stretch of road in the plane(x, t).

Scénario 3: Reduction of capacity

The demand at the input of the network is set to 0.3056veh/s and the supply at the output is set to 0.2222veh/s. The initial concentration of cells is set at 50veh/km.

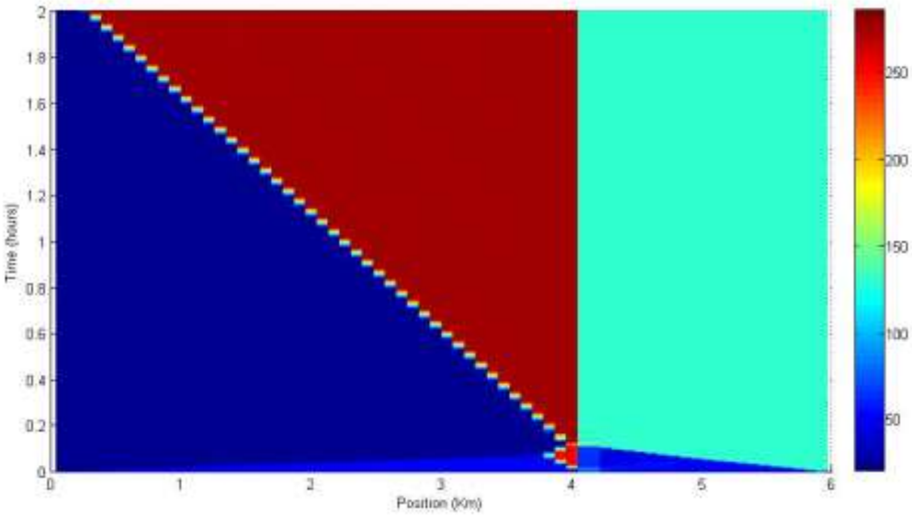

Fig. 12: Results of the simulation with the hybrid model in the case of a reduced capacity

Through the results presented in figure 12, we observe that the hybrid model provides an acceptable representation of the traffic, with a representation of the traffic evolution at the discontinuity point, keeping the flow conservation and considering the speed as a basic parameter at the discontinuity point. The simulation allowed to observe the evolution of the rise in congestion to upstream that is observable in reality.

CONCLUSION

In this paper we presented a hybrid traffic flow modeling based on the macroscopic LWR model and a microscopic traffic flow model developed independently.

The interest of coupling the two models is to represent a large network with a macroscopic first order model, and using a microscopic model to represent the singular elements.

The results show that with this hybrid model it is able to obtain acceptable results for a representation of the different characteristics of traffic. This hybrid model can also represent phenomena that can be observed at the points of spatial discontinuities, and that are not represented by the macroscopic models.

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