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Research Article

Trial Equation Method for Exact Travelling Solutions of Fifth Order Caudrey-Dodd-Gibbon Equation

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Abstract: Under the travelling wave transformation, some nonlinear partial differential equations such as the fifth order Caudrey-Dodd-Gibbon equation are transformed to ordinary differential equation. Then Using trial equation method and combing complete discrimination system for polynomial, the classifications of all single traveling wave solution to this equation are obtained.
Keywords: the nonlinear partial differential equation; complete discrimination system for polynomial; trial equation method; traveling wave transform; the fifth order Caudrey-Dodd-Gibbon equation

INTRODUCTION

Many problems in natural and engineering sciences are modeled by partial differential equations(PDE).To find solutions of PDE is a very important problem. Many mathematicians and physicists work in the field, and they have developed many methods for many special equations. Particularly, by algebrac expansion method, many exact solutions of many nonlinear equations have been obtained. Recently, Professor Liu proposed a powerful method named trial equation method[1-3] for finding exact solutions to nonlinear differential equations.

The Caudrey-Dodd-Gibbon equation's physical understanding was illustrated in[4],and its solitary solutions have been studied by many authors[5-10].It's worth mentioning that Wazwaz derived explicit travelling wave solutions using the tank method in2006 and multiple-soliton solutions using Hirota's direct method combined with the simplified Hereman method in 2008 for the above equation. In this paper, we mainly use Liu's trial equation method and the theory of complete discrimination system for polynomial[11-15] to solve exact solutions of the fifth order Caudrey-Dodd-Gibbon equation.

DESCRIPTION OF TRIAL EQUATION METHOD

The objective of this section is to outline the use of trial equation method for solving a nonlinear PDE. Suppose we have a nonlinear PDE for $u(x,t)$,in the form

$$P(u,u_x,u_t,u_{xx},u_{tt},u_{xt},\cdots)=0 \tag{1}$$

where P is a polynomial, which includes nonlinear terms and the highest order derivatives and so on.

Step 1. Taking the wave transformation $u=u(\xi_1),\xi_1=kx+\omega t$, reduces Eq.(1) to the ordinary differential equation (ODE).

$$M(u,ku',\omega u',k^2u'',\omega^2u'',k\omega u'',\cdots)=0 \tag{2}$$

Step 2. Take trial equation method

$$u''=F(u)=a_0+a_1u+\cdots+a_mu^m. \tag{3}$$

Integrating the Equation (3) with respect to ξ_1 once, we get

$$(u')^2=H(u)=\frac{2a_m}{m+1}u^{m+1}+\cdots+a_1u^2+2a_0u+d \tag{4}$$

where m, a_i and integration constant d are to be determined. Substituting Eqs. $(3),(4)$ and other derivative terms into Eq.(2) yields a polynomial $G(u)$ of u. According to the balance principle we can determine the value of m. Setting the coefficients of $G(u)$ to zero, we get a system of algebraic equations. Solving this system, we can determine values of $a_o, a_1, \cdots, a_m$ and integration constant.

Step 3. Rewrite Eq.(4) by the integral form

$$\pm(\xi - \xi_0) = \int \frac{du}{\sqrt{H(u)}}. \tag{5}$$

According to the complete discrimination system of the polynomial, we classify the roots of $H(u)$ and solve the integral equation (5). Thus we obtain the exact solutions to Eq.(1).

APPLICATION OF TRIAL EQUATION METHOD

The fifth order Caudrey-Dodd-Gibbon equation reads as

$$u_t + 30u_x u_{xx} + 30uu_{xxx} + 180u^2 u_x + u_{xxxxx} = 0. \tag{6}$$

Taking the traveling wave transformation $u = u(\xi_1)$ and $\xi_1 = kx + \omega t$, we can obtain the corresponding reduced ODE.

$$\omega u' + 30k^3 u'u'' + 30k^3 uu''' + 180ku^2 u' + k^5 u''''' = 0. \tag{7}$$

Integrating Eq.(7) once with respect to ξ_1 and setting the integration constant as zero yields

$$\omega u + 30k^3 uu'' + 60ku^3 + k^5 u'''' = 0. \tag{8}$$

We take the trial equation as follows

$$u'' = a_0 + a_1 u + \cdots + a_m u^m. \tag{9}$$

According to the trial equation method of rank homogeneous equation, balancing u'''' (or uu'') with u^3 gets $m = 2$, so Eq.(9) has the following specific form

$$u'' = a_0 + a_1 u + a_2 u^2. \tag{10}$$

Integrating Eq.(10) with respect to ξ_1 once, we yield

$$(u')^2 = \frac{2}{3} a_2 u^3 + a_1 u^2 + 2a_0 u + d. \tag{11}$$

where values of a_o, a_1, a_2 and the integration constant d are to be determined latter. By Eqs.(10) and (11),we can derive the following formula

$$u'''' = \frac{10}{3} a_2^2 u^3 + 5a_1 a_2 u^2 + (6a_0 a_2 + a_1^2)u + a_0 a_1 + 2a_2 d. \tag{12}$$

Substituting Eqs.(10) and (12) into Eq.(8), we have

$$r_3 u^3 + r_2 u^2 + r_1 u + r_0 = 0. \tag{13}$$

where

$$r_0 = k^5 a_0 a_1 + 2k^5 a_2 d. \tag{14}$$

$$r_1 = \omega + 30k^3 a_0 + k^5(6a_0 a_2 + a_1^2). \tag{15}$$

$$r_2 = 30k^3 a_1 + 5k^5 a_1 a_2. \tag{16}$$

$$r_3 = 30k^3 a_2 + 60k + \frac{10}{3} k^5 a_2^2. \tag{17}$$

Let the coefficient $r_i = 0 (i = 0,1,2,3)$ be zero, we will yield nonlinear algebraic equations. Solving the equations, we will determine the values of a_0, a_1, a_2, d. We get two groups of solution to the equations as follows

$$a_0 = -\frac{\omega}{12k^3}, a_1 = 0, a_2 = -\frac{3}{k^2}, d = 0 \tag{18}$$

$$a_0 = \frac{\omega + k^5 a_1^2}{6k^3}, a_1 = a_1, a_2 = -\frac{6}{k^2}, d = \frac{(\omega + k^5 a_1^2)a_1}{72k} \tag{19}$$

Where a_1 is an arbitrary constant.

When the above condition (18) or (19) is satisfied, we use the complete discrimination system for the third order polynomial and have the following solving process.

Let

$$v=(\frac{2}{3}a_2)^{\frac{1}{3}}u, \xi=(\frac{2}{3}a_2)^{\frac{1}{3}}\xi_1, d_2=a_1(\frac{2}{3}a_2)^{-\frac{2}{3}}, d_1=2a_0(\frac{2}{3}a_2)^{-\frac{1}{3}}, d_0=d. \quad (20)$$

Then Eq.(11) becomes

$$(v')^2=v^3+d_2v^2+d_1v+d_0. \quad (21)$$

Where v is a function of ξ. The integral form of Eq.(21) is

$$\pm(\xi-\xi_0)=\int\frac{dv}{\sqrt{v^3+d_2v^2+d_1v+d_0}}. \quad (22)$$

Denote

$$F(v)=v^3+d_2v^2+d_1v+d_0. \quad (23)$$

$$\Delta=-27(\frac{2d_2^3}{27}+d_0-\frac{d_1d_3}{3})^2-4(d_1-d_2^2)^3, D_1=d_1-\frac{d_2^2}{3}. \quad (24)$$

According to the complete discrimination system, we give the corresponding single traveling wave solutions to Eq.(6).

Case1. $\Delta=0, D_1<0. F(v)=0$ has a double real root and a simple real root. Then we have

$$F(v)=(v-\lambda_1)^2(v-\lambda_2), \lambda_1\neq\lambda_2. \quad (25)$$

When $v>\lambda_2$, the corresponding solutions are

$$u_1=(\frac{2}{3}a_2)^{-\frac{1}{3}}\{(\lambda_1-\lambda_2)\tanh^2[\frac{\sqrt{\lambda_1-\lambda_2}}{2}(\frac{2}{3}a_2)^{\frac{1}{3}}(kx+\omega t-\xi_0)]+\lambda_2\}, (\lambda_1>\lambda_2); \quad (26)$$

$$u_2=(\frac{2}{3}a_2)^{-\frac{1}{3}}\{(\lambda_1-\lambda_2)\coth^2[\frac{\sqrt{\lambda_1-\lambda_2}}{2}(\frac{2}{3}a_2)^{\frac{1}{3}}(kx+\omega t-\xi_0)]+\lambda_2\}, (\lambda_1>\lambda_2); \quad (27)$$

$$u_3=(\frac{2}{3}a_2)^{-\frac{1}{3}}\{(-\lambda_1+\lambda_2)\sec^2[\frac{\sqrt{-\lambda_1+\lambda_2}}{2}(\frac{2}{3}a_2)^{\frac{1}{3}}(kx+\omega t-\xi_0)]+\lambda_1\}, (\lambda_1<\lambda_2). \quad (28)$$

Case2. $\Delta=0, D_1=0. F(v)=0$ has a triple root. Then we have

$$F(v)=(v-\lambda)^3. \quad (29)$$

The corresponding solution is

$$u_4=4(\frac{2}{3}a_2)^{-\frac{2}{3}}(kx+\omega t-\xi_0)^{-2}+\lambda. \quad (30)$$

Case 3. $\Delta>0, D_1<0. F(v)=0$ has three different real roots. Then we have

$$F(v)=(v-\lambda_1)(v-\lambda_2)(v-\lambda_3), \lambda_1<\lambda_2<\lambda_3. \quad (31)$$

When $\lambda_1<v<\lambda_2$, we take the transformation as follows

$$v=\lambda_1+(\lambda_2-\lambda_1)sin^2\phi. \quad (32)$$

According to the Eq.(19), we have

$$\pm(\xi-\xi_0)=\int\frac{dv}{\sqrt{F(v)}}=\frac{2}{\sqrt{\lambda_3-\lambda_1}}\int\frac{d\phi}{\sqrt{1-m^2sin^2\phi}}. \quad (33)$$

Where $m^2=\frac{\lambda_2-\lambda_1}{\lambda_3-\lambda_1}$. On the basis of Eq.(30) and the definition of the Jacobi elliptic sine function, we have

$$v=\lambda_1+(\lambda_2-\lambda_1)\,\mathrm{sn}^2\left(\frac{\sqrt{\lambda_3-\lambda_1}}{2}(\xi_1-\xi_0), m\right). \quad (34)$$

The corresponding solutions is

$$u_5=(\frac{2}{3}a_2)^{-\frac{1}{3}}\left[\lambda_1+(\lambda_2-\lambda_1)\,\mathrm{sn}^2\left(\frac{\sqrt{\lambda_3-\lambda_1}}{2}(\frac{2}{3}a_2)^{\frac{1}{3}}(kx+\omega t-\xi_0), m\right)\right]. \quad (35)$$

When $v > \lambda_3$, we take the transformation as follows

$$v = \frac{-\lambda_2 \sin^2\phi + \lambda_3}{\cos^2\phi}. \tag{36}$$

The corresponding solutions is

$$u_6 = (\frac{2}{3}a_2)^{-\frac{1}{3}} \left[\frac{\lambda_3 - \lambda_2 \operatorname{sn}^2\left(\frac{\sqrt{\lambda_3-\lambda_1}}{2}(\frac{2}{3}a_2)^{\frac{1}{3}}(kx+\omega t-\xi_0), m\right)}{\operatorname{cn}^2\left(\frac{\sqrt{\lambda_3-\lambda_1}}{2}(\frac{2}{3}a_2)^{\frac{1}{3}}(kx+\omega t-\xi_0), m\right)} \right]. \tag{37}$$

where $m^2 = \frac{\lambda_2-\lambda_1}{\lambda_3-\lambda_1}$.

Case4. $\Delta < 0. F(v) = 0$ has only a real root. Then we have

$$F(v) = (v-\lambda)(v^2 + pv + q),\ p^2 - 4q < 0. \tag{38}$$

When $v > \lambda_1$, we take the transformation as follows

$$v = \lambda + \sqrt{\lambda^2 + p\lambda + q}\tan^2\frac{\phi}{2}. \tag{39}$$

According to the Eq.(19), we have

$$\xi - \xi_0 = \int \frac{dv}{\sqrt{(v-\lambda)(v^2+pv+q)}} = \frac{1}{(\lambda^2+p\lambda+q)^{\frac{1}{4}}} \int \frac{d\phi}{\sqrt{1-m^2 sin^2\phi}}. \tag{40}$$

where $m^2 = \frac{1}{2}\left(1 - \frac{\lambda+\frac{p}{2}}{\sqrt{\lambda^2+p\lambda+q}}\right)$. On the basis of Eq.(37) and the definition of the Jacobi elliptic cosine function, we have

$$v = \lambda + \frac{2\sqrt{\lambda^2+p\lambda+q}}{1+\operatorname{cn}\left((\lambda^2+p\lambda+q)^{\frac{1}{4}}(\xi_1-\xi_0), m\right)} - \sqrt{\lambda^2+p\lambda+q}. \tag{41}$$

The corresponding solutions is

$$u_7 = (\frac{2}{3}a_2)^{-\frac{1}{3}} \left[\lambda + \frac{2\sqrt{\lambda^2+p\lambda+q}}{1+\operatorname{cn}\left((\lambda^2+p\lambda+q)^{\frac{1}{4}}(\frac{2}{3}a_2)^{\frac{1}{3}}(kx+\omega t-\xi_0), m\right)} - \sqrt{\lambda^2+p\lambda+q} \right]. \tag{42}$$

In Eqs.(26)(27)(28)(30)(35)(37) and (42), the integration constant ξ_0 has been rewritten, but we still use it. The solutions $u_i (i = 1, \cdots, 7)$ are all possible exact traveling wave solutions to Eq.(6). We can see it is easy to write the corresponding solutions to the fifth order Caudrey-Dodd-Gibbon equation.

CONCLUSION

Trial equation method is a systematic method to solve nonlinear differential equations. The advantage of this method is that we can deal with nonlinear equations with linear methods. This method has the characteristics of simple steps and clear effectivity. Based on the idea of the trial equation method and the aid of the computerized symbolic computation, some exact traveling wave solutions to the fifth order Caudrey-Dodd-Gibbon equation have been obtained.With the same method, some of other equations can be dealt with.

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