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Research Article

Solving a problem of road lighting with Matlab language

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Abstract: In this paper, the problem of road lighting is considered systematically by using Matlab language. Firstly, we make the problem of road lighting abstraction to a coordinate axis point, line, surface, and increasing the number of independent variables to study the dependent variable light intensity. The technique is based on nonlinear differential equation with Matlab derivation study it's extreme. Finally, this paper also extends the road around the corner and three-dimensional illumination intensity on both sides of the road situation.
Keywords: Matlab, Road lighting problem, Extremum

Point set of lighting intensity on a road

In this article, we abstract the problem of street lighting to a coordinate axis based on methodology [1-3]. X -axis is a horizontal road of the streetlights and the opposite are the two longitudinal axes of op_1 and sp_2, where p_i is the brightness of the lights, h_i is the height of the lights. The coordinates of the two lights are $(0, h_1)$ and (s, h_2), where s is the horizontal distance between the two lights. Make some point between the two on the road be $X = (x, 0)$. We will find the point with the minimum lighting intensity. The horizontal distance between X and the first light is x while the distance between X and the second light is $s - x$. So the distance between X and the two light sources, which is r_1 and r_2 respectively, are

$$r_1^2 = h_1^2 + x^2, \quad r_2^2 = h_2^2 + (s-x)^2.$$

Light intensity of the two lights from the X point is

$$I_1(x) = \frac{p_1}{r_1^2} = \frac{p_1}{h_1^2 + x^2}, \quad I_2(x) = \frac{p_2}{r_2^2} = \frac{p_2}{h_2^2 + (s-x)^2}.$$

If the projection angle of lights are α_1 and α_2 respectively, conditions of the lights also depend on $\sin\alpha_1$ and $\sin\alpha_2$

$$\sin\alpha_1 = \frac{h_1}{\sqrt{h_1^2 + x^2}}, \quad \sin\alpha_2 = \frac{h_2}{\sqrt{h_2^2 + (s-x)^2}}.$$

So the total lighting intensity of point X is

$$I(x) = I_1(x)\sin\alpha_1 + I_2(x)\sin\alpha_2 = \frac{p_1 h_1}{\sqrt{(h_1^2 + x^2)^3}} + \frac{p_2 h_2}{\sqrt{(h_2^2 + (s-x)^2)^3}}. \qquad (1)$$

Specifically

$$I(x) = \frac{10}{\sqrt{(25 + x^2)^3}} + \frac{18}{\sqrt{(36 + (20-x)^2)^3}}. \qquad (2)$$

According to (1) formula, we can know

$$I'(x) = \frac{-3p_1 h_1 x}{(h_1^2 + x^2)^{5/2}} - \frac{3p_2 h_2(-2s + 2x)}{2(h_2^2 + (s-x)^2)^{5/2}} = 0. \qquad (3)$$

Make p_1 = 2000 [w], p_2 = 3000 [w], h_1 = 5 [m], h_2 = 6 [m], s = 20 [m], and we can know

$$I'(x) = \frac{-30x}{\sqrt{(25+x^2)^5}} + \frac{54(20-x)}{\sqrt{(36+(20-x)^2)^5}} . \qquad (4)$$

We can use Matlab to calculate the root of the formula: $I'(x) = 0$ (Table 1)

Table 1 Value of $I(x)$ when x is in the interval of [0, 20]

x	0	0.028489970	9.3382991	19.976695	20
$I(x)$	0.08197716	0.08198104	0.01824393	0.08447655	0.08447468

We can see: when x = 9.338m, it is the darkest point; when x = 19.977m, it is the brightest point.

Change h_2 to maximize lighting intensity

We use the same values in the previous section, but we take the height of the second light source as a variable h_2 in order to maximize the lighting intensity of X . So, $I(x, h_2)$ are a function of two variables and the height of the streetlights with 3kW power can change between 3m and 9m.Therefore, the lighting intensity of point Q is a binary function on x and h_2 :

$$I(x,h_2) = \frac{p_1 h_1}{\sqrt{(h_1^2 + x^2)^3}} + \frac{p_2 h_2}{\sqrt{(h_2^2 + (s-x)^2)^3}} = \frac{10}{\sqrt{(25+x^2)^3}} + \frac{3h_2}{\sqrt{(h_2^2 + (20-x)^2)^3}}$$

Similarly, we can figure out the extremes of the function $I(x, h_2)$ .And they are the darkest point and the brightest point respectively.

$$\frac{\partial I}{\partial h_2} = \frac{p_2}{\sqrt{(h_2^2 + (s-x)^2)^3}} - \frac{3 p_2 h_2^2}{\sqrt{(h_2^2 + (s-x)^2)^5}} = 0 \qquad (5)$$

Make $h_2 = h$ and by using Matlab, we can know: x_1 = 20 +2 ^ (1/2) * h (rounding), x_2 = 20-2 ^ (1/2) * h.

$$\frac{\partial I}{\partial x} = \frac{-3 p_1 h_1 x}{\sqrt{(h_1^2 + x^2)^5}} + \frac{3 p_2 h_2 (s-x)}{\sqrt{(h_2^2 + (s-x)^2)^5}} = \frac{-30(20-\sqrt{2}h_2)}{\sqrt{(25+x^2)^5}} + \frac{9h_2(20-x)}{\sqrt{(h_2^2 + (20-x)^2)^5}} = 0 \quad (6)$$

By Matlab, we can figure out h_2 = 7.42239m. And we can further figure out x and the Brightness I ,i.e.when x = 9.5032 and h_2 = 7.42239, the darkest point of maximum brightness is 0.0186w.

The extreme lighting intensity of the corner of the road

Next, we are going to promote the problem of road lighting. In order to be more realistic, we assume that we need to place a safety warning sign near the corner of the road. Obviously, streetlights and the corner point is not on the same plane. For the convenience of calculation, we abstract the curved corner to an obtuse angle of 120 ° and the links, between the streetlights on both sides, forms an isosceles triangle with the obtuse angle being 120 °. p_i is the lighting intensity of streetlights, h_i is the height of the lights, s is the horizontal distance between the two lights.

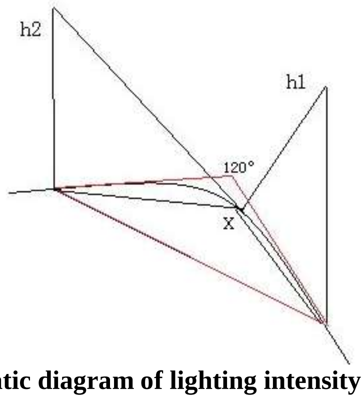

Fig- 1: Schematic diagram of lighting intensity of the corner

Make some point, between the two lights on the road, be $X = (x, 0, 0)$, For simplicity, we assume that

p_1=2000[w], p_2=3000[w], h_1=5[m], h_2=6[m], $s = 10\sqrt{2}$ [m],

According to the law of cosine, we know $b = \frac{\sqrt{2}}{2}s$, $c = \frac{\sqrt{2}}{2}s - x$, so $s = 120 - 11s$, thus,

$$I(x) = I_1(x)\sin\alpha_1 + I_2(x)\sin\alpha_2 = \frac{p_1 h_1}{\sqrt{(h_1^2 + x^2)^3}} + \frac{p_2 h_2}{\sqrt{(h_2^2 + (120 - 11x)^2)^3}}. \quad (7)$$

Take the specific values into the formula and minimize $I(x)$, then we can get the coordinate of point X. We can calculate the extreme point of the function. We can make derivation of $I(x)$ and figure out its roots. Then we use Matlab to calculate the corresponding values (Table 2).

Table-2 : Corresponding values of $I(x)$ when x takes four points in the interval [0, 10]

x	0	0.00029656941	8.9668854	10
$I(x)$	80.010377725881	80.0103781480125	10.8879935197022	18.5045886036065

That is, when x = 8.967m, it is the darkest point, when x = 0.000297m, it is the brightest point. According to the symmetry of interval [0, 10], we can see that there are corresponding points on the other side, being the darkest and the brightest point of the corner respectively. From a practical perspective, it is suggested that a safety warning sign be placed at the distance of 0.000297m from the light h_1 in the direction of h_2.

Extreme area of lighting intensity on the sides of the three-dimensional road

This section extends the lighting intensity of three-dimensional road. In order to be more realistic, we assume streetlights are placed on the two sides of the road. We abstract the road to the bottom of the cube; streetlights are abstracted to the four heights. If the height of streetlight is h_i, then the corresponding lighting intensity of street is p_i. Obviously, streetlights and road can be seen as a cube, where s is the horizontal distance between the two lights. Make some point between the two on the road be $X = (x, 0, 0)$. In order to facilitate the calculation, we assume: p_1 = 1000 [w], p_2 = 2000 [w], p_3 = 3000 [w], p_4 = 4000 [w], h_1 = 6 [m], h_2 = 5 [m], h_3 = 4 [m], h_4 = 3 [m], s = 10 [m].

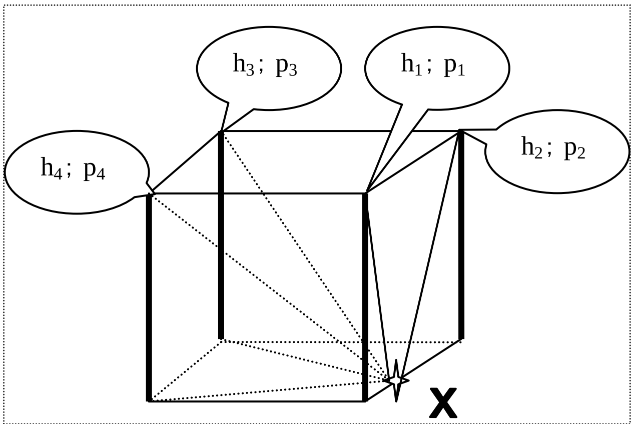

Fig-2: Schematic diagram of the lighting of three-dimensional road

$$I(x) = I_1(x)\sin\alpha_1 + I_2(x)\sin\alpha_2 + I_3(x)\sin\alpha_3 + I_4(x)\sin\alpha_4$$

$$I(x) = \frac{p_1 h_1}{\sqrt{(h_1^2 + x^2)^3}} + \frac{p_2 h_2}{\sqrt{(h_2^2 + (10 - x)^2)^3}} + \frac{p_3 h_3}{\sqrt{(h_3^2 + (10 - x)^2 + s^2)^3}} + \frac{p_4 h_4}{\sqrt{(h_4^2 + x^2 + s^2)^3}}. \quad (8)$$

Take the value into the formula and make derivation of $I(x)$, we can get $I'(x)$. As x is between [0, 10], we can figure out zero points of the function in the interval [0， 10] by dichotomy. After selecting a valid x, we can calculate the corresponding values of $I(x)$ by Matlab (Table 3).

Table-3:Corresponding values of $I(x)$ when x takes three points in the interval [0, 10]

x	0	9.85290527	10
$I(x)$	49.25814860387617	97.46304219752855	97.35955460993185

Thus, when x = 0m, it is the darkest point; when x = 9.853m, it is the brightest point.

References

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