Research Article

Special family of Diophantine Triples
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Abstract: In this paper, we present two special Diophantine triples in which the sum of any two is a perfect square.

Keywords: Integral solutions, non-extendable diophantine triples

INTRODUCTION:
A set of positive integers \( \{a_1, a_2, \ldots, a_m\} \) is said to have the property D(n), \( n \in \mathbb{Z} \setminus \{0\} \), if \( a_i a_j + n \) is a perfect square for all \( 1 \leq i < j \leq m \) and such a set is called a Diophantine m-tuple with property D(n) or a \( P_n \) set of size m were studied by Diophantus [1]. For an extensive review of various article one may refer [2-15]. Many mathematicians considered the problem of the existence of Diophantine quadruples with the property D(n) for any arbitrary integer n and also for any linear polynomials in n.

In this communication we present two special Diophantine triples such that in each case the sum of any two is a perfect square

METHOD ANALYSIS
Construction of Diophantine triples:
Let \( a = r^2 + s^2, b = 2rs \) (\( r > s > 0 \)) be any two integers such that \( (a + b) \) is a perfect square. We search for a distinct integer \( c \) such that

\[ a + c = \alpha^2 \]
\[ b + c = \beta^2 \]

(1) \( \rightarrow \)
\[ \alpha^2 - \beta^2 = (r - s)^2 \]

(2) \( \rightarrow \)

(3)

Choice: I
Choose \( r \) and \( s \) in (3) such that
\[ r - s = P^2 - Q^2 \]

and thus
\[ \alpha = P^2 + Q^2 \text{ and } \beta = 2PQ \]

Substituting the value of \( \alpha \) in (1) or value of \( \beta \) in (2), we get,
\[ c = (P^2 + Q^2)^2 - a \text{ or } c = (2PQ)^2 - b \]
The above process is illustrated below in Table I

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>P</th>
<th>Q</th>
<th>a</th>
<th>b</th>
<th>c</th>
</tr>
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<tbody>
<tr>
<td>15</td>
<td>7</td>
<td>1</td>
<td>274</td>
<td>210</td>
<td>-174</td>
<td></td>
</tr>
<tr>
<td>22</td>
<td>10</td>
<td>2</td>
<td>584</td>
<td>440</td>
<td>-184</td>
<td></td>
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<tr>
<td>24</td>
<td>11</td>
<td>6</td>
<td>697</td>
<td>528</td>
<td>6528</td>
<td></td>
</tr>
<tr>
<td>26</td>
<td>11</td>
<td>7</td>
<td>797</td>
<td>572</td>
<td>11972</td>
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<tr>
<td>32</td>
<td>15</td>
<td>8</td>
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<td>960</td>
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<tr>
<td>42</td>
<td>21</td>
<td>2</td>
<td>2205</td>
<td>1764</td>
<td>-1364</td>
<td></td>
</tr>
<tr>
<td>42</td>
<td>21</td>
<td>10</td>
<td>2205</td>
<td>1764</td>
<td>46636</td>
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</tr>
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</table>

Note that all the triples in the table are all non-extendable triples except the triple (17,8,8) which can be extended to a quadruple (17,8,8,8).

Choice: II
Choose r and s so that
\[ r - s = 2PQ \quad (P > Q) \]

Then,
\[ \alpha = P^2 + Q^2, \beta = P^2 - Q^2 \]

Substituting the values of \( \alpha \) in (1) or \( \beta \) in (2), we get,
\[ c = \alpha^2 - a(r,s) \quad \text{(or)} \quad c = \beta^2 - b(r,s) \]

The above process is illustrated below in Table II

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>P</th>
<th>Q</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>Remark</th>
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<td>2</td>
<td>290</td>
<td>34</td>
<td>110</td>
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<td></td>
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<tr>
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<td>4</td>
<td>2</td>
<td>416</td>
<td>160</td>
<td>-16</td>
<td>Non-extendable triple</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>3</td>
<td>2</td>
<td>234</td>
<td>90</td>
<td>-65</td>
<td>Non-extendable triple</td>
<td></td>
</tr>
<tr>
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<td>2</td>
<td>3</td>
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<td></td>
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<td>46636</td>
<td>Non-extendable triple</td>
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</table>

CONCLUSION
In this paper, we have presented some non-extendable Diophantine triples. One may search for extendable Diophantine triples consisting of special numbers.

2000 Mathematics subject classification number: 11D09.

REFERENCES
12. Yasutsurgu Fujita, Alain Togbe, Uniqueness of the extension of the \(D(4k^2)\) triple \([k^2-4,k^2,4k^2-4]\) NNTDM, 2011; 17 :442-449.