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Research Article

Some special non-extendable Diophantine Triples

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Abstract: We construct some non-extendable P_{-3} Diophantine triples with property and exhibited few theorems on P_{-3}

Keywords: Integral solutions, non-extendable, Diophantine triples.

INTRODUCTION

A set of positive integers $(a_1, a_2, \ldots a_m\}$ is said to have the property D(n), $n \in Z - \{0\}$, if $a_i a_j + n$ is a perfect square for all $1 \le i < j \le m$ and such a set is called a Diophantine m-tuple with property D(n) or a P_n set of size m. The problem of construction of such set was studied by Diophantus. He studied the following problem. Find four (positive rational) numbers such that the product of any two of them increased by one is a perfect square. He obtained the following solution $\frac{1}{16}, \frac{33}{16}, \frac{17}{4}, \frac{115}{16}$ .The first set of four positive integers with the above property was found by Fermat and it was {1,3,8,120}.Euler gave the solution $\{a, b, a+b+2r, 4r(r+a)(r+b)\}$ where $ab+1 = r^2$ For an extensive review of various articles one may refer [2-15].

METHODS ANALYSIS

In this communication we construct some non-extendable P_{-3}, P_{-7}, P_{-8} and P_{-12} sets and exhibited some theorems on these sets

Theorem:1 $(1,4,7)$ is P_{-3} non-extendable set.

Proof: Assume that n is a positive integer such that the set $(1,4,7)$ can be extended. Let us find x,y,z such that

$$n - 3 = x^2 \qquad (1)$$

$$4n - 3 = y^2 \qquad (2)$$

$$7n - 3 = z^2 \qquad (3)$$

Eliminating n between (1) and (2),we get,

$$y^2 - 4x^2 = 9$$

which has only four integer solutions namely $(\pm 5, \pm 2), (\pm 5, \mp 2)$

Substituting the x-value in (1) or the y-value in (2), we have $n = 7$ which does not satisfy (3).Hence $(1,4,7)$ is P_{-3} non-extendable set.

Theorem:2 The set P_{-3} does not include any positive multiple of 5.

Proof: Let d be an element of the set P_{-3}.If $5k(k \in Z)$ is also an element of the set P_{-3},then

$$5kd - 3 = x^2$$

By taking modulo 5 on both sides of the equation

$$x^2 \equiv 2(\text{mod}\, 5) \qquad (4)$$

Employing Euler criterion note that (4) is insolvable. Thus, P_{-3} does not include any multiple of 5.

Similarly, we can prove the set P_{-3} does not include any multiple of 11.

Cor: Some non-extendable Diophantine triples are presented below.

Sets	Properties	Remarks
(1,16,176)	P_{-7}	Does not contain positive multiple of 5.
(1,16,21)	P_{-12}	Does not include positive multiple of 5
(1,9,12)	P_{-8}	Does not include positive multiples of 5 and 11

CONCLUSION

In this paper, we have presented some non-extendable Diophantine triples. One may search for extendable Diophantine triples consisting of special numbers.

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